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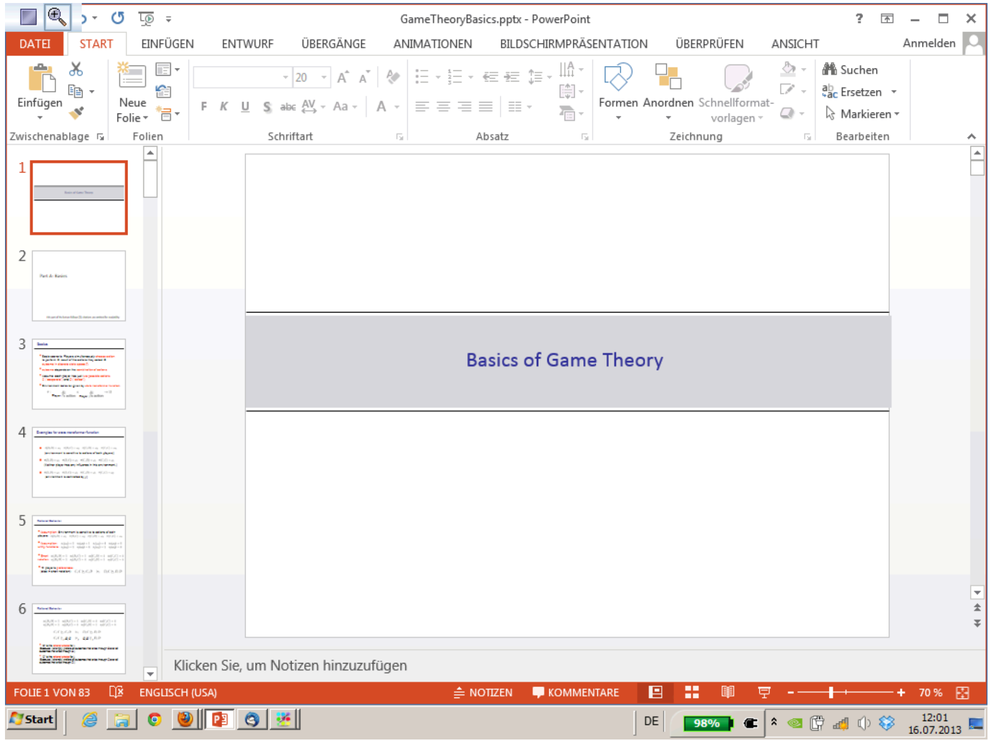

Basics

- Basic scenario: Players simultaneously choose action to perform → result of the actions they select → outcome in discrete state space Ω
- outcome depends on the *combination* of actions
- Assume: each player has just two possible actions *C* (“cooperate”) and *D* (“defect”)
- Environment behavior given by *state transformer function*:

$$\tau : \underbrace{Ac}_{\text{Player } i\text{'s action}} \times \underbrace{Ac}_{\text{Player } j\text{'s action}} \to \Omega$$

Examples for state transformer function

- $\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_3 \quad \tau(C,C) = \omega_4$
  (environment is sensitive to actions of both players)
- $\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_1 \quad \tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_1$
  (Neither player has any influence in this environment.)
- $\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_2$
  (environment is controlled by *j*.)

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- $\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_2$
  (environment is controlled by *j*.)

Rational Behavior

- Assumption: Environment is sensitive to actions of both players: $\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_3 \quad \tau(C,C) = \omega_4$
- Assumption: Utility functions: $u_i(\omega_1) = 1 \quad u_i(\omega_2) = 1 \quad u_i(\omega_3) = 4 \quad u_i(\omega_4) = 4$
  $u_j(\omega_1) = 1 \quad u_j(\omega_2) = 4 \quad u_j(\omega_3) = 1 \quad u_j(\omega_4) = 4$
- Short notation: $u_i(D,D) = 1 \quad u_i(D,C) = 1 \quad u_i(C,D) = 4 \quad u_i(C,C) = 4$
  $u_j(D,D) = 1 \quad u_j(D,C) = 4 \quad u_j(C,D) = 1 \quad u_j(C,C) = 4$
- → player's preferences:
  (also in short notation): $C,C \succeq_i C,D \quad \succ_i \quad D,C \succeq_i D,D$

Rational Behavior

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- Short notation: $u_i(D,D) = 1 \quad u_i(D,C) = 1 \quad u_i(C,D) = 4 \quad u_i(C,C) = 4$
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- → player's preferences:
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Rational Behavior

$$u_i(D,D)=1 \quad u_i(D,C)=1 \quad u_i(C,D)=4 \quad u_i(C,C)=4$$
$$u_j(D,D)=1 \quad u_j(D,C)=4 \quad u_j(C,D)=1 \quad u_j(C,C)=4$$

$$C,C \succeq_i C,D \quad \succ_i \quad D,C \succeq_i D,D$$
$$C,C \succeq_j D,C \quad \succ_j \quad C,D \succeq_j D,D$$

- "*C*" is the *rational choice* for i.
(Because i (strongly) prefers all outcomes that arise through *C* over all outcomes that arise through *D*.)
- "*C*" is the *rational choice* for j.
(Because j (strongly) prefers all outcomes that arise through *C* over all outcomes that arise through *D*.)

Rational Behavior

- Game theory: characterize the previous scenario in a *payoff matrix*:

i

		defect	coop
j	defect	1 / 1	4 / 1
	coop	1 / 4	4 / 4

same as:
$$u_i(D,D)=1 \quad u_i(D,C)=1 \quad u_i(C,D)=4 \quad u_i(C,C)=4$$
$$u_j(D,D)=1 \quad u_j(D,C)=4 \quad u_j(C,D)=1 \quad u_j(C,C)=4$$

- Player i is "*column player*"
- Player j is "*row player*"

Dominant Strategies and Nash Equilibria

- With respect to „what should I do":
If $\Omega = \Omega_1 \cup \Omega_2$ we say „Ω_1 weakly dominates Ω_2 for player i"
iff for player i every state (outcome) in Ω_1 is preferable to or at least as good as every state in Ω_2:
$$\forall \omega_1 \forall \omega_2 : (\omega_1 \in \Omega_1 \wedge \omega_2 \in \Omega_2) \rightarrow \omega_1 \succeq_i \omega_2$$
- If $\Omega = \Omega_1 \cup \Omega_2$ we say „Ω_1 strongly dominates Ω_2 for player i"
iff for player i every state (outcome) in Ω_1 is preferable to every state in Ω_2:
$$\forall \omega_1 \forall \omega_2 : (\omega_1 \in \Omega_1 \wedge \omega_2 \in \Omega_2) \rightarrow \omega_1 \succ \omega_2$$
- Example:
$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\} \qquad \omega_1 \succ_i \omega_2 \succ_i \omega_3 \succ_i \omega_4$$
$$\left.\begin{array}{l}\Omega_1 = \{\omega_1, \omega_2\} \\ \Omega_2 = \{\omega_3, \omega_4\}\end{array}\right\} \text{„}\Omega_1 \text{ strongly dominates } \Omega_2 \text{ for player i" :}$$

Dominant Strategies and Nash Equilibria

- Game theory notation: actions are called „strategies"
- Notation: s^* is the set of possible outcomes (states) when „playing strategy s" (executing action s)
- Example: if we have (as before):
$$\tau(D,D)=\omega_1 \quad \tau(D,C)=\omega_2 \quad \tau(C,D)=\omega_3 \quad \tau(C,C)=\omega_4$$
we have (from player i's point of view):
$$D^* = \{\omega_1, \omega_2\} \qquad C^* = \{\omega_3, \omega_4\}$$
- Notation: „strategy $s1$ (strongly / weakly) dominates $s2$" iff $s1^*$ (strongly / weakly) dominates $s2^*$
- If one strategy strongly dominates the other → question what to do is easy. (do first)

The Prisoner's Dilemma

- Two criminals are held in separate cells (no communication):

  (1) One confesses and the other does not → confessor is freed and the other gets 3 years

  (2) Both confess → each gets 2 years

  (3) Neither confesses → both get 1 year

- Associations: Confess == D; Not Confess == C
- Payoff matrix

	i defects	i cooperates
j defects	2 / 2	0 / 5
j cooperates	5 / 0	3 / 3

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The Prisoner's Dilemma

	i:D	i:C
j:D	2 / 2	0 / 5
j:C	5 / 0	3 / 3

$$u_i(D,D)=2,\quad u_i(D,C)=5,\quad u_i(C,D)=0,\quad u_i(C,C)=3$$
$$u_j(D,D)=2,\quad u_j(D,C)=0,\quad u_j(C,D)=5,\quad u_j(C,C)=3$$

$$(D,C)\succ_i(C,C)\succ_i(D,D)\succ_i(C,D)$$
$$(C,D)\succ_j(C,C)\succ_j(D,D)\succ_j(D,C)$$

- Take place of prisoner (e.g. prisoner i) →
  Course of Reasoning:
  - suppose I cooperate: If j also cooperates → we both get payoff 3. If j defects → I get payoff 0. ➔ Best guaranteed payoff when I cooperate is 0
  - suppose I defect: If j cooperates → I get payoff 5. If j also defects → both get payoff 2. ➔ Best guaranteed payoff when I defect is 2
  - ➔ If I defect I'll get a minimum guaranteed payoff of 2. If I cooperate I'll get a minimum guaranteed payoff of 0.
  - ➔ If prefer guaranteed payoff of 2 to guaranteed payoff of 0.

    ➔ I should defect

The Prisoner‘s Dilemma

	i:D	i:C
j:D	2 / 2	5 / 0
j:C	0 / 5	3 / 3

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  - ➔ If I defect I‘ll get a minimum guaranteed payoff of 2. If I cooperate I‘ll get a minimum guaranteed payoff of 0.
  - ➔ If prefer guaranteed payoff of 2 to guaranteed payoff of 0.

    ➔ I should defect

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$$(C,D)\succ_j(C,C)\succ_j(D,D)\succ_j(D,C)$$

- only one Nash equilibrium: (D,D). („under the assumption that the other does D, one can do no better than do D“)
- Intuition says: (C,C) is better than (D,D) so why not (C,C)? → but if player assumes that other player does C it is BEST to do D! → seemingly „waste of utility“
- „shocking“ truth: defect is rational, cooperate is irrational
- Other prisoner‘s dilemma: Nuclear arms reduction (D: do not reduce, C: reduce)

The Prisoner‘s Dilemma

	i:D	i:C
j:D	2 / 2	5 / 0
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„sucker‘s payoff“

Other symmetric 2x2 Games

Stag Hunt

- Going back to J.J.Russeau (1775)
- Modern variant: You and a friend decide: good joke to appear both naked on a party. C: really do it; D: not do it

$$(C,C) \succ_i (D,C) \succ_i (D,D) \succ_i (C,D)$$

	i:D	i:C
j:D	1, 1	2, 0
j:C	0, 2	3, 3

- Two Nash equilibria: (D,D), (C,C)

(Assuming the other does D you can do no better than do D
Assuming the other does C you can do no better than do C)

Other symmetric 2x2 Games

Game of Chicken

- Going back to a James Dean film
- Modern variant: Gangster and hero drive cars directly towards each other C: steer away; D: not steer away

$$(D,C) \succ_i (C,C) \succ_i (C,D) \succ_i (D,D)$$

	i:D	i:C
j:D	0, 0	3, 1
j:C	1, 3	2, 2

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Assuming the other does C you can do no better than do D)

Notation: Strategic Form Games

- Set $\mathcal{I}$ of players: {1,2,...,I}
  Example: {1,2}
- Player index: $i \in \mathcal{I}$
- Pure Strategy Space S_i of player i
  Example: S_1={U,M,D} and S_2={L,M,R}
- Stragegy profile $s=(s_1,\dots,s_I)$ where each $s_i \in S_i$
  Example: (D,M)
- (Finite) space $S = \times_i S_i$ of strategy profiles $s \in S$
  Example: S = { (U,L), (U,M),..., (D,R) }
- Payoff function $u_i: S \rightarrow \mathbb{R}$ gives von Neumann-Morgenstern-utility $u_i(s)$ for player i of strategy profile $s \in S$
  Examples: u_1((U,L))=4 , u_2((U,L))=3 , u_1((M,M))=8 .....
- Set of player i's opponents: „-i“
  Example: -1={2}

	L	M	R
U	4 , 3	5 , 1	6 , 2
M	2 , 1	8 , 4	3 , 6
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Example: -1={2}

Notation: Strategic Form Games

- Two Player zero sum game: $\forall s: \sum_{i=1}^{2} u_i(s) = 0$

- Structure of game is common knowledge:
all players know;
all players know that all players know;
all players know that all players know that all players know;
....

- Mixed strategy $\sigma_i : S_i \rightarrow [0,1]$ Probability distribution over pure strategies (statistically independent for each player);
Examples: $\sigma_1(U)=1/3$, $\sigma_1(M)=2/3$, $\sigma_1(D)=0$;
$\sigma'_1(U)=2/3$, $\sigma'_1(M)=1/6$, $\sigma'_1(D)=1/6$;
.....

- Thus: $\sigma_i(s_i)$ is the probability that player i assigns to strategy (action) s_i

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Sense of Mixed Strategy Concept

- Example: Rock Paper Scissors

	Rock	Paper	Scissors
Rock	0 , 0	-1 , 1	1 , -1
Paper	1 , -1	0 , 0	-1 , 1
Scissors	-1 , 1	1 , -1	0 , 0

- no pure NE, but mixed NE if both play (1/3, 1/3, 1/3)

Notation: Strategic Form Games

- Space of mixed strategies for player i: Σ_i
- Space of mixed strategy profiles: $\Sigma = \times_i \Sigma_i$
- Mixed strategy profile $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_I) \in \Sigma$
- Player i's payoff when a mixed strategy profile σ is played is

$$\sum_{s\in S}\left(\prod_{j=1}^{I}\sigma_j(s_j)\right)u_i(s)$$

denoted as $u_i(\sigma)$, is a linear function of the σ_i

- A pure strategy of a player is a special mixed strategy of that player with one probability equal to 1 and all others equal to 0

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- Mixed strategy profile $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_I) \in \Sigma$
- Player i's payoff when a mixed strategy profile σ is played is

$$\sum_{s\in S}\left(\prod_{j=1}^{I}\sigma_j(s_j)\right)u_i(s)$$

denoted as $u_i(\sigma)$, is a linear function of the σ_i

- A pure strategy of a player is a special mixed strategy of that player with one probability equal to 1 and all others equal to 0

Notation: Strategic Form Games

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Notation: Strategic Form Games

Example:

Let

$\sigma_1(U)=1/3$, $\sigma_1(M)=1/3$, $\sigma_1(D)=1/3$
$\sigma_2(L)=0$, $\sigma_2(M)=1/2$, $\sigma_2(R)=1/2$

or short

$\sigma_1 = (1/3, 1/3, 1/3)$
$\sigma_2 = (0, 1/2, 1/2)$

	L	M	R
U	4 , 3	5 , 1	6 , 2
M	2 , 1	8 , 4	3 , 6
D	3 , 0	9 , 6	2 , 8

We then have:

$$u_1(\sigma_1, \sigma_2) = 1/3\,(0*4 + ½*5 + ½*6) + 1/3\,(0*2 + ½*8 + ½*3) + 1/3\,(0*3 + ½*9 + ½*2) = 11/2$$

$$u_2(\sigma_1, \sigma_2) = \ldots = 27/6$$

Games in Strategic Form & Nash Equilibrium

- What is rational to do?
- No matter what player 1 does: R gives player 2 a strictly higher payoff than M. „M is strictly dominated by R“
- → player 1 knows that player 2 will not play M → U is better than M or D
- → player 2 knows that player 1 knows that player 2 will not play M → player 2 knows that player 1 will play U → player 2 will play L
- This elimination process: „iterated strict dominance“

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- New example:
- Player 1: M not dominated by U and M not dominated by D
- But: If Player 1 plays σ_1 =(1/2, 0, 1/2) he will get u(σ_1)=1/2 regardless how player 2 plays
- → a pure strategy may be dominated by a mixed strategy even if it is not strictly dominated by any pure strategy

	L	R
U	2, 0	-1, 0
M	0, 0	0, 0
D	-1, 0	2, 0