Script generated by TTT

Title: Seidl: Virtual_Machines (03.07.2012)

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Pages: 52

Example:

$$
\begin{array}{lll}
\text{bigger}(X,Y) & \leftarrow & X = \textit{elephant}, Y = \textit{horse} \\
\text{bigger}(X,Y) & \leftarrow & X = \textit{horse}, Y = \textit{donkey} \\
\text{bigger}(X,Y) & \leftarrow & X = \textit{donkey}, Y = \textit{dog} \\
\text{bigger}(X,Y) & \leftarrow & X = \textit{donkey}, Y = \textit{monkey} \\
\text{is\_bigger}(X,Y) & \leftarrow & \text{bigger}(X,Y) \\
\text{is\_bigger}(X,Y) & \leftarrow & \text{bigger}(X,Z), \text{is\_bigger}(Z,Y) \\
? & \text{is\_bigger}(\textit{elephant}, \textit{dog}) &
\end{array}
$$

A More Realistic Example:

$$
\begin{array}{lll}
\text{app}(X,Y,Z) & \leftarrow & X = [\,],\; Y = Z \\
\text{app}(X,Y,Z) & \leftarrow & X = [H|X'],\; Z = [H|Z'],\; \text{app}(X',Y,Z') \\
? & \text{app}(X, [Y,c], [a,b,Z]) &
\end{array}
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\end{array}
$$

Remark:

$[\,]$	==	the atom empty list
$[H\|Z]$	==	binary constructor application
$[a,b,Z]$	==	shortcut for: $[a\|[b\|[Z\|[\,]]]]$

A program p is constructed as follows:

$$
\begin{array}{lcl}
t & ::= & a \mid X \mid \_ \mid f(t_1, \ldots, t_n) \\
g & ::= & p(t_1, \ldots, t_k) \mid X = t \\
c & ::= & p(X_1, \ldots, X_k) \leftarrow g_1, \ldots, g_r \\
p & ::= & c_1 . \ldots . c_m ? g
\end{array}
$$

- A term t either is an atom, a variable, an anonymous variable or a constructor application.
- A goal g either is a literal, i.e., a predicate call, or a unification.
- A clause c consists of a head $p(X_1, \ldots, X_k)$ with predicate name and list of formal parameters together with a body, i.e., a sequence of goals.
- A program consists of a sequence of clauses together with a single goal as query.

Procedural View of Proll programs:

~~goal~~	$==$	procedure call
predicate	$==$	procedure
clause	$==$	definition
term	$==$	value
unification	$==$	basic computation step
binding of variables	$==$	side effect

Note: Predicate calls ...

- ... do not have a return value.
- ... affect the caller through side effects only :-)
- ... may fail. Then the next definition is tried :-))

$\Longrightarrow$ backtracking

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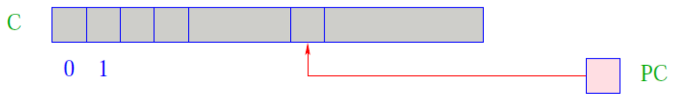
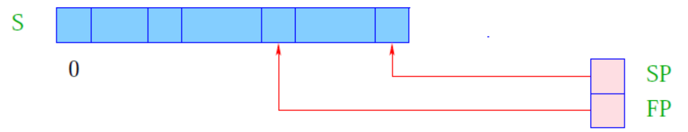
27 Architecture of the WiM:

The Code Store:



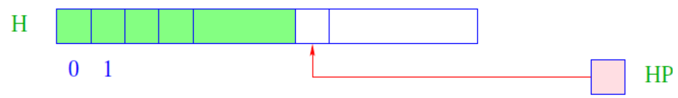
C	=	Code store – contains WiM program; every cell contains one instruction;
PC	=	Program Counter – points to the next instruction to executed;

The Runtime Stack:



S	=	Runtime Stack – every cell may contain a value or an address;
SP	=	Stack Pointer – points to the topmost occupied cell;
FP	=	Frame Pointer – points to the current stack frame.
		Frames are created for predicate calls,
		contain cells for each variable of the current clause

The Heap:



H	=	Heap for dynamicly constructed terms;
HP	=	Heap-Pointer – points to the first free cell;

- The heap in maintained like a stack as well :-)
- A new-instruction allocates a object in H.
- Objects are tagged with their types (as in the MaMa) ...

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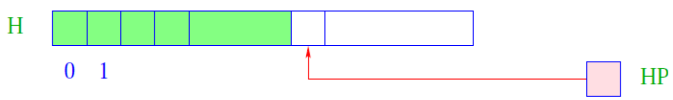

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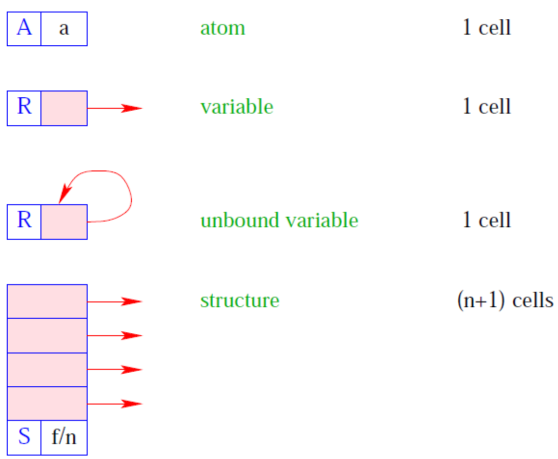


28 Construction of Terms in the Heap

Parameter terms of goals (calls) are constructed in the heap before passing.

Assume that the address environment ρ returns, for each clause variable X its address (relative to FP) on the stack. Then $\text{code}_A\ t\ \rho$ should ...

- construct (a presentation of) t in the heap; and
- return a reference to it on top of the stack.

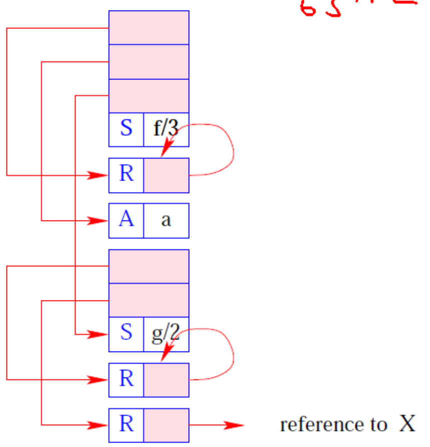
Idea:

- Construct the tree during a post-order traversal of t
- with one instruction for each new node!

Example: $t \equiv f(g(X, Y), a, Z)$.

Assume that X is initialized, i.e., $S[FP + \rho\, X]$ contains already a reference, Y and Z are not yet initialized.

Representing $t \equiv f(g(X, Y), a, Z)$:



For a distinction, we mark occurrences of already initialized variables through over-lining (e.g. $\bar{X}$).

Note: Arguments are always initialized!

Then we define:

$$
\begin{array}{lll}
\text{code}_A\ a\ \rho & = & \text{putatom a} \\
\text{code}_A\ X\ \rho & = & \text{putvar}\ (\rho\, X) \\
\text{code}_A\ \bar{X}\ \rho & = & \text{putref}\ (\rho\, X) \\
\text{code}_A\ \_\ \rho & = & \text{putanon}
\end{array}
\qquad
\begin{array}{lll}
\text{code}_A\ f(t_1, \ldots, t_n)\ \rho & = & \text{code}_A\ t_1\ \rho \\
 & & \ldots \\
 & & \text{code}_A\ t_n\ \rho \\
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For $f(g(\overline{X}, Y), a, Z)$ and $\rho = \{X \mapsto 1, Y \mapsto 2, Z \mapsto 3\}$ this results in the sequence:

putref 1	putatom a
putvar 2	putvar 3
putstruct g/2	putstruct f/3

$p(X) \leftarrow q(\bar{X}, Y).$

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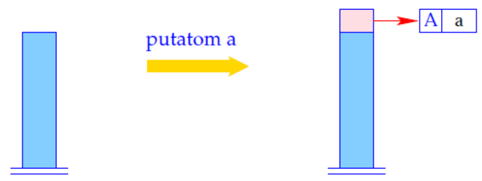
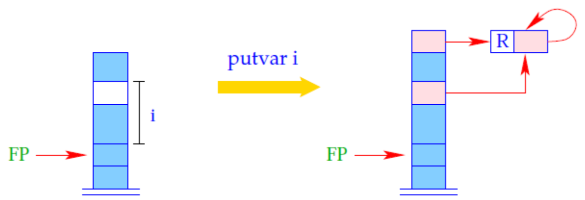
The instruction putatom a constructs an atom in the heap:



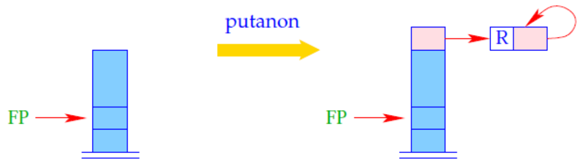
SP++; S[SP] = new (A,a);

The instruction putvar i introduces a new unbound variable and additionally initializes the corresponding cell in the stack frame:



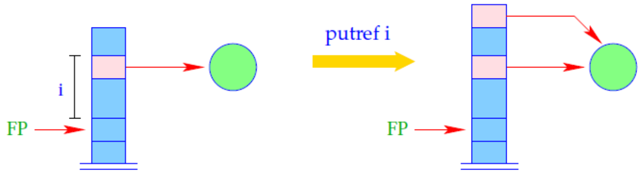
SP = SP + 1;
S[SP] = new (R, HP);
S[FP + i] = S[SP];

The instruction putanon introduces a new unbound variable but does not store a reference to it in the stack frame:

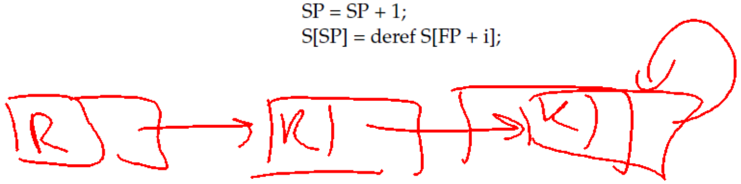


SP = SP + 1;
S[SP] = new (R, HP);

The instruction putref i pushes the value of the variable onto the stack:



SP = SP + 1;
S[SP] = deref S[FP + i];



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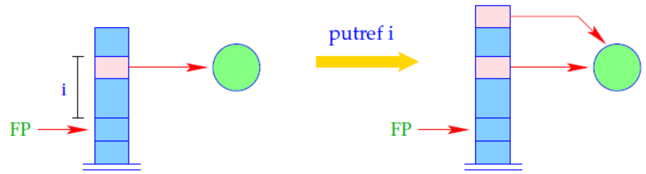
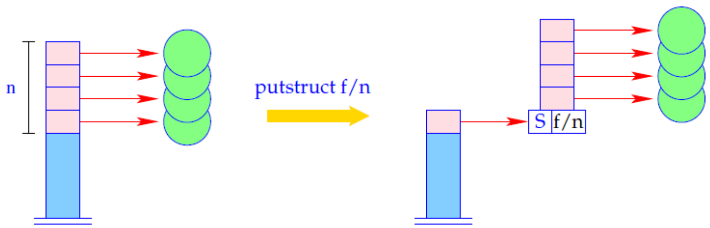


SP = SP + 1;
S[SP] = deref S[FP + i];

The auxiliary function deref contracts chains of references:

```
ref deref (ref v) {
    if (H[v]==(R,w) && v!=w) return deref (w);
    else return v;
}
```


The instruction putstruct f/n builds a constructor application in the heap:



v = new (S, f, n);
SP = SP - n + 1;
for (i=1; i<=n; i++)
  H[v + i] = S[SP + i -1];
S[SP] = v;

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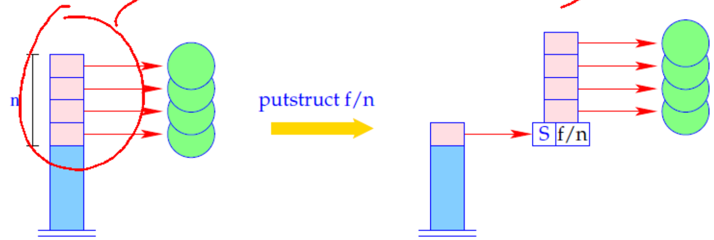

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for (i=1; i<=n; i++)
  H[v + i] = S[SP + i -1];
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29 The Translation of Literals (Goals)

Idea:

- Literals are treated as procedure calls.
- We first allocate a stack frame.
- Then we construct the actual parameters (in the heap)
- ... and store references to these into the stack frame.
- Finally, we jump to the code for the procedure/predicate.

$\text{code}_G\ p(t_1, \ldots, t_k)\ \rho\ =$

```
        mark B          // allocates the stack frame
        codeA t1 ρ
        ...
        codeA tk ρ
        call p/k        // calls the procedure p/k
B :     ...
```


$\text{code}_G\ p(t_1, \ldots, t_k)\ \rho\ =$

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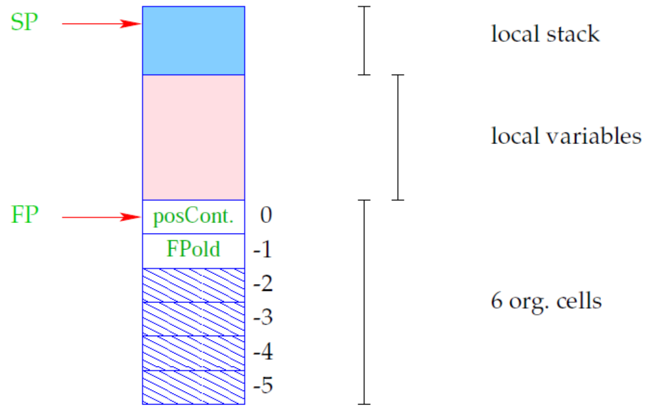
B :     ...
```

Example: $p(a, X, g(\bar{X}, Y))$ with $\rho = \{X \mapsto 1, Y \mapsto 2\}$

We obtain:

mark B	putref 1		call p/3
putatom a	putvar 2	B:	...
putvar 1	putstruct g/2		

Stack Frame of the WiM:

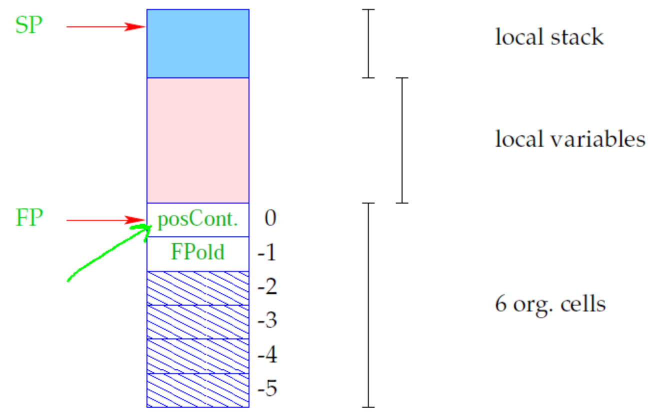


Remarks:

- The positive continuation address records where to continue after successful treatment of the goal.
- Additional organizational cells are needed for the implementation of backtracking

⟹ will be discussed at the translation of predicates.

Stack Frame of the WiM:



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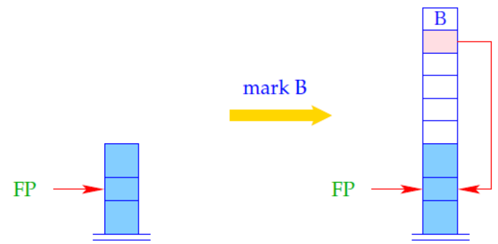
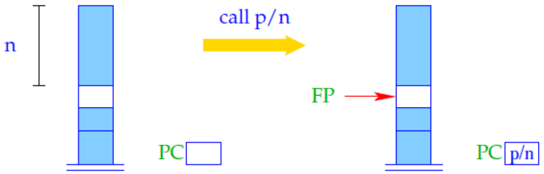
The instruction mark B allocates a new stack frame:



SP = SP + 6;
S[SP] = B; S[SP-1] = FP;

The instruction call p/n calls the n-ary predicate p :



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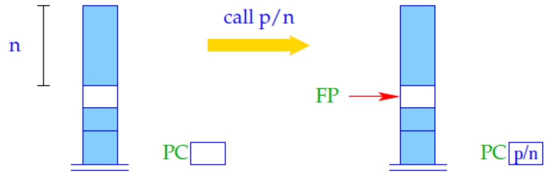

FP = SP - n;
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30 Unification

Convention:

- By $\bar{X}$, we denote an occurrence of X;
  it will be translated differently depending on whether the variable is initialized or not.
- We introduce the macro put $\tilde{X}$ ρ :

$$
\begin{aligned}
\text{put } X\ \rho &= \text{putvar } (\rho\ X) \\
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X = a

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Let us translate the unification $\tilde{X} = t$.

Idea 1:

- Push a reference to (the binding of) X onto the stack;
- Construct the term t in the heap;
- Invent a new instruction implementing the unification :-)

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Example:

Consider the equation:

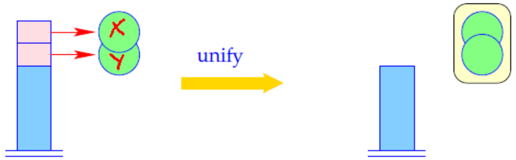
$$\bar{U} = f(g(\bar{X}, Y), a, Z)$$

Then we obtain for an address environment

$$\rho = \{X \mapsto 1, Y \mapsto 2, Z \mapsto 3, U \mapsto 4\}$$

```
putref 4    putref 1        putatom a       unify
            putvar 2        putvar 3
            putstruct g/2   putstruct f/3
```


The instruction unify calls the run-time function `unify()` for the topmost two references:




```
unify (S[SP-1], S[SP]);
SP = SP–2;
```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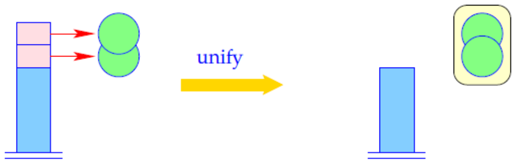

unify (S[SP-1], S[SP]);
SP = SP–2;

X = f(X)

```
bool unify (ref u, ref v) {
   if (u == v) return true;
   if (H[u] == (R,_)) {
      if (H[v] == (R,_)) {
         if (u>v) {
            H[u] = (R,v); trail (u); return true;
         } else {
            H[v] = (R,u); trail (v); return true;
         }
      } elseif (check (u,v)) {
         H[u] = (R,v); trail (u); return true;
      } else {
         backtrack(); return false;
      }
   }
   ...
```

```
...
if ((H[v] == (R,_)) {
    if (check (v,u)) {
        H[v] = (R,u); trail (v); return true;
    } else {
        backtrack(); return false;
    }
  }
  if (H[u]==(A,a) && H[v]==(A,a))
    return true;
  if (H[u]==(S, f/n) && H[v]==(S, f/n)) {
    for (int i=1; i<=n; i++)
      if(!unify (deref (H[u+i]), deref (H[v+i])) return false;
    return true;
  }
  backtrack(); return false;
}
```



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    if (check (v,u)) {
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  }
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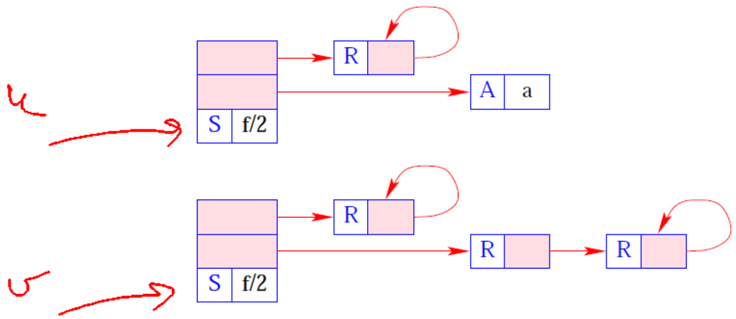

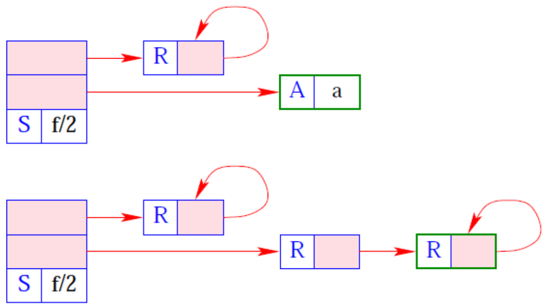

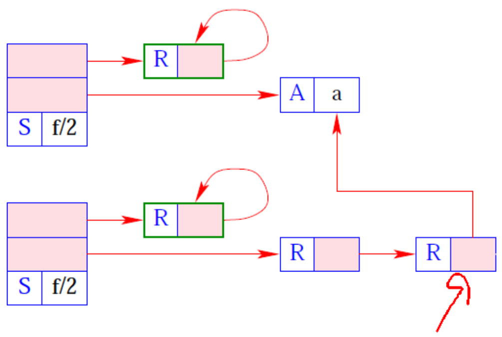

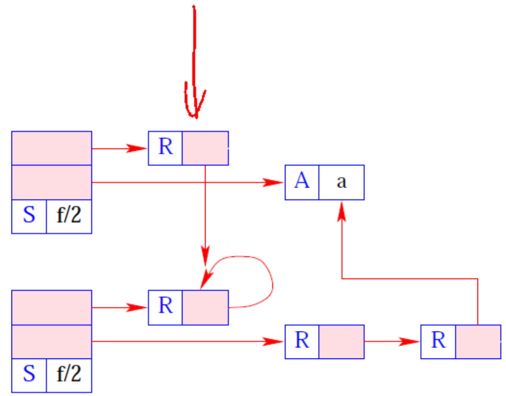

- The run-time function `trail()` records the ~~potential~~ new binding.
- The run-time function `backtrack()` initiates backtracking.
- The auxiliary function `check()` performs the occur-check: it tests whether a variable (the first argument) occurs inside a term (the second argument).
- Often, this check is skipped, i.e.,

```
bool check (ref u, ref v) { return true;}
```


Otherwise, we could implement the run-time function `check()` as follows:

```
bool check (ref u, ref v) {
   if (u == v) return false;
   if (H[v] == (S, f/n)) {
      for (int i=1; i<=n; i++)
         if (!check(u, deref (H[v+i])))
            return false;
   return true;
}
```

Discussion:

- The translation of an equation $\tilde{X} = t$ is very simple :-)
- Often the constructed cells immediately become garbage :-(

Idea 2:

- Push a reference to the run-time binding of the left-hand side onto the stack.
- Avoid to construct sub-terms of t whenever possible !
- Translate each node of t into an instruction which performs the unifcation with this node !!