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Social Gaming / Social Computing SS 2015

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Examples for state transformer function

- $\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_3 \quad \tau(C,C) = \omega_4$
  (environment is sensitive to actions of both players)
- $\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_1 \quad \tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_1$
  (Neither player has any influence in this environment.)
- $\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_2$
  (environment is controlled by *j*.)

Rational Behavior

- **Assumption:** Environment is sensitive to actions of both players: $\tau(D,D)=\omega_1 \quad \tau(D,C)=\omega_2 \quad \tau(C,D)=\omega_3 \quad \tau(C,C)=\omega_4$
- **Assumption: Utility functions:** $u_i(\omega_1)=1 \quad u_i(\omega_2)=1 \quad u_i(\omega_3)=4 \quad u_i(\omega_4)=4$ $u_j(\omega_1)=1 \quad u_j(\omega_2)=4 \quad u_j(\omega_3)=1 \quad u_j(\omega_4)=4$
- **Short notation:** $u_i(D,D)=1 \quad u_i(D,C)=1 \quad u_i(C,D)=4 \quad u_i(C,C)=4$ $u_j(D,D)=1 \quad u_j(D,C)=4 \quad u_j(C,D)=1 \quad u_j(C,C)=4$
- → player's preferences: (also in short notation): $C,C \succeq_i C,D \quad \succ_i \quad D,C \succeq_i D,D$

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Rational Behavior

- Game theory: characterize the previous scenario in a *payoff matrix*:

		i: defect	i: coop
j	defect	1 1	4 1
	coop	1 4	4 4

same as: $u_i(D,D)=1 \quad u_i(D,C)=1 \quad u_i(C,D)=4 \quad u_i(C,C)=4$ $u_j(D,D)=1 \quad u_j(D,C)=4 \quad u_j(C,D)=1 \quad u_j(C,C)=4$

- Player i is "*column player*"
- Player j is "*row player*"

Rational Behavior

- Game theory: characterize the previous scenario in a *payoff matrix*:

		i: defect	*i*: coop
j	defect	$u_i = 1$, $u_j = 1$	$u_i = 4$, $u_j = 1$
j	coop	$u_i = 1$, $u_j = 4$	$u_i = 4$, $u_j = 4$

same as: $u_i(D,D)=1 \quad u_i(D,C)=1 \quad u_i(C,D)=4 \quad u_i(C,C)=4$
$u_j(D,D)=1 \quad u_j(D,C)=4 \quad u_j(C,D)=1 \quad u_j(C,C)=4$

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Dominant Strategies and Nash Equilibria

- With respect to „what should I do":

If $\Omega = \Omega_1 \cup \Omega_2$ we say „Ω_1 weakly dominates Ω_2 for player i" iff for player i every state (outcome) in Ω_1 is preferable to or at least as good as every state in Ω_2:

$$\forall \omega_1 \forall \omega_2 : (\omega_1 \in \Omega_1 \land \omega_2 \in \Omega_2) \rightarrow \omega_1 \succeq_i \omega_2$$

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$$\forall \omega_1 \forall \omega_2 : (\omega_1 \in \Omega_1 \land \omega_2 \in \Omega_2) \rightarrow \omega_1 \succ \omega_2$$

- Example:

$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ $\quad \Omega_1 = \{\omega_1, \omega_2\}$

$\omega_1 \succ_i \omega_2 \succ_i \omega_3 \succ_i \omega_4$ $\quad \Omega_2 = \{\omega_3, \omega_4\}$

„Ω_1 strongly dominates Ω_2 for player i" :

Dominant Strategies and Nash Equilibria

- Game theory notation: actions are called „strategies"
- Notation: s^* is the set of possible outcomes (states) when „playing strategy s" (executing action s)
- Example: if we have (as before):

$$\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_3 \quad \tau(C,C) = \omega_4$$

we have (from player i's point of view):

$$D^* = \{\omega_1, \omega_2\} \qquad C^* = \{\omega_3, \omega_4\}$$

- Notation: „strategy $s1$ (strongly / weakly) dominates $s2$" iff $s1^*$ (strongly / weakly) dominates $s2^*$
- If one strategy strongly dominates the other → question what to do is easy. (do first)

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Dominant Strategies and Nash Equilibria

- → delete strongly dominated strategies from options
- Two strategies of two players are in „**Nash equilibrium**" iff

  (1) Assuming that player i plays s1, player j can do no better than play s2
  (2) Assuming that player j plays s2, player i can do no better than play s1

→ In a Nash equilibrium, no player has an incentive to deviate from strategy of equilibrium.

Competitive and Zero-Sum Interactions

- Scenario („**strictly competitive**"): Player i prefers outcome ω over ω' iff player j prefers outcome ω' over ω :

$$\omega \succ_i \omega' \leftrightarrow \omega' \succ_j \omega$$

- Scenario („**zero-sum**"):

$$\forall \omega \in \Omega: \; u_i(\omega)+u_j(\omega)=0$$

- zero-sum games are always strictly competitive
- zero-sum games imply negative utility for „loser"
- strictly zero-sum: only in games like chess. Real world never „strictly zero-sum" (Example: two girls compete to win the heart of the same guy). But: Unfortunately many encounters are perceived as zero sum games.

The Prisoner's Dilemma

- Two criminals are held in separate cells (no communication):

  (1) One confesses and the other does not → confessor is freed and the other gets 3 years
  (2) Both confess → each gets 2 years
  (3) Neither confesses → both get 1 year

- Associations: Confess == D; Not Confess == C
- Payoff matrix

	i defects	i cooperates
j defects	2 (j), 2 (i)	5 (j), 0 (i)
j cooperates	0 (j), 5 (i)	3 (j), 3 (i)

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	i defects	i cooperates
j defects	2 / 2	0 / 5
j cooperates	5 / 0	3 / 3

The Prisoner's Dilemma

	i:D	i:C
j:D	2 / 2	0 / 5
j:C	5 / 0	3 / 3

$$u_i(D,D)=2,\quad u_i(D,C)=5,\quad u_i(C,D)=0,\quad u_i(C,C)=3$$
$$u_j(D,D)=2,\quad u_j(D,C)=0,\quad u_j(C,D)=5,\quad u_j(C,C)=3$$

$$(D,C)\succ_i(C,C)\succ_i(D,D)\succ_i(C,D)$$
$$(C,D)\succ_j(C,C)\succ_j(D,D)\succ_j(D,C)$$

- Take place of prisoner (e.g. prisoner i) → **Course of Reasoning**:
  - suppose I cooperate: If j also cooperates → we both get payoff 3. If j defects → I get payoff 0. ➔ Best guaranteed payoff when I cooperate is 0
  - suppose I defect: If j cooperates → I get payoff 5. If j also defects → both get payoff 2. ➔ Best guaranteed payoff when I defect is 2
  - ➔ If I defect I'll get a minimum guaranteed payoff of 2. If I cooperate I'll get a minimum guaranteed payoff of 0.
  - ➔ If prefer guaranteed payoff of 2 to guaranteed payoff of 0.
  - ➔ I should defect

The Prisoner's Dilemma

- „Defect more rational than cooperate" → Humans: Machiavellism (opposed to real altruism)
- Philosophical question: isn't even altruism ultimately some kind of optimization towards OWN goals?!
- Further aspect: Strict rationalism (in case of prisoner's dilemma: defect) is usually only applied when sucker's payoff really hurts.
- What we have not yet regarded: Multiple sequential games between same players → „The shadow of the future" → What does it mean for rationalism and strategy?

The shadow of the future: Iterated Prisoner's Dilemma Game

- Game is played multiple times. Players can see all past actions of other player.
- Course of reasoning:
  - If I defect, the other player my punish me by defecting in the next run. (not a point in the one shot Prisoner's Dilemma game)
  - Testing cooperation (and possibly getting the sucker's payoff) is not tragic, because „on the long run" one (or several) sucker's payoff(s) is (are) „statistically" not important (can e.g. be equaled by gains through mutual cooperation)
- → in an iterated PD-game: cooperation is rational

Competing PD-strategies: Axelrod's tournament (1980)

Some strategies competing:

a note on wording: here we use the word strategy in its usual sense; not the former notation strategy == action

- ALL-D: Always defect
- RANDOM: Choose D or C randomly
- TIT-FOR-TAT: On the first round cooperate; on round t do what opponent did on round t-1
- TESTER: Intention: Exploit „nice" programs that not punish D: On first round, test opponent with D. If opponent retaliates with D → play TIT-FOR-TAT. If opponent does C → play CCD.
- JOSS: like TESTER but with p=0.1 deviate from CCD by replacing a C with D

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- Winner was TIT-FOR-TAT (5 lines of Fortran code).
- Why? Overall Score of a strategy computed as average of performance against **all** other strategies. (TIT-FOR-TAT was defeated by ALL-D).
- TIT-FOR-THAT won against „cooperative strategies" → Again (as in analysis of single PD): being not too cooperative pays out.
- → Axelrod distilled some rules from outcome of tournament:

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Competing PD-strategies: Axelrod's tournament (1980)

(1) Do not be envious: Not necessary to „beat" opponent to do well

(2) Do not be first to defect: Cooperation is risky (sucker's payoff) but overall, some losses do not count that much and cooperation may result in win-win-situations (C,C)

(3) Reciprocate C and D: TIT-FOR-TAT balances punishing and forgiving → encourages cooperation for other player. TIT-FOR-TAT is fair: retaliates exactly with the same amount of maliciousness as opponent

(4) Don't be too clever: TIT-FOR-TAT was simplest but won over programs with complex models of opponent's strategies:

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- Reasons why lack of opponent model was beneficial:

(1) In course of tournament, models are not improved by learning (respecting reciprocal learning of other players)

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(3) Complex model without learning → same as random

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Other symmetric 2x2 Games

- „2x2“: two players, each with two actions;
  Symmetric:

$$(D,C) \succ_i (C,C) \succ_i (D,D) \succ_i (C,D)$$
$$(C,D) \succ_j (C,C) \succ_j (D,D) \succ_j (D,C)$$

symmetry axis

- Other symmetric 2x2 games (There are 4!=24 such games):

$(D,C) \succ_i (C,C) \succ_i (D,D) \succ_i (C,D)$	Prisoner‘s Dilemma
$(D,C) \succ_i (C,C) \succ_i (C,D) \succ_i (D,D)$	Game of Chicken
$(C,C) \succ_i (D,C) \succ_i (D,D) \succ_i (C,D)$	Stag Hunt
$(D,D) \succ_i (D,C) \succ_i (C,D) \succ_i (C,C)$	Defection dominates
$(D,D) \succ_i (D,C) \succ_i (C,C) \succ_i (C,D)$	Defection dominates
$(C,C) \succ_i (C,D) \succ_i (D,C) \succ_i (D,D)$	Cooperation dominates
$(C,C) \succ_i (C,D) \succ_i (D,D) \succ_i (D,C)$	Cooperation dominates

Other symmetric 2x2 Games

Stag Hunt

- Going back to J.J.Russeau (1775)
- Modern variant: You and a friend decide: good joke to appear both naked on a party. C: really do it; D: not do it

$$(C,C) \succ_i (D,C) \succ_i (D,D) \succ_i (C,D)$$

	i:D	i:C
j:D	1 / 1	0 / 2
j:C	2 / 0	3 / 3

- Two Nash equilibria: (D,D), (C,C)

(Assuming the other does D you can do no better than do D
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Game of Chicken

- Going back to a James Dean film
- Modern variant: Gangster and hero drive cars directly towards each other C: steer away; D: not steer away

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Notation: Strategic Form Games

- Set $\mathcal{I}$ of players: {1,2,...,I}

Example: {1,2}

- Player index: $i \in \mathcal{I}$
- Pure Strategy Space S_i of player i

Example: S_1={U,M,D} and S_2={L,M,R}

- Stragegy profile $s=(s_1,\dots s_I)$ where each $s_i \in S_i$

Example: (D,M)

	L	M	R
U	4 , 3	5 , 1	6 , 2
M	2 , 1	8 , 4	3 , 6
D	3 , 0	9 , 6	2 , 8

- (Finite) space $S = \times_i S_i$ of strategy profiles $s \in S$

Example: S = { (U,L), (U,M),..., (D,R) }

- Payoff function u_i: $S \rightarrow \mathbb{R}$ gives von Neumann-Morgenstern-utility $u_i(s)$ for player i of strategy profile $s \in S$

Examples: $u_1((U,L))=4$, $u_2((U,L))=3$, $u_1((M,M))=8$ .....

- Set of player i's opponents: „-i"

Example: -1={2}

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Notation: Strategic Form Games

- Two Player zero sum game: $\forall s : \sum_{i=1}^{2} u_i(s) = 0$

- Structure of game is common knowledge:
all players know;
all players know that all players know;
all players know that all players know that all players know;
….

- Mixed strategy $\sigma_i : S_i \rightarrow [0,1]$ Probability distribution over pure strategies (statistically independent for each player);
Examples: $\sigma_1(U)=1/3$, $\sigma_1(M)=2/3$, $\sigma_1(D)=0$;
$\sigma'_1(U)=2/3$, $\sigma'_1(M)=1/6$, $\sigma'_1(D)=1/6$;
…..

- Thus: $\sigma_i(s_i)$ is the probability that player i assigns to strategy (action) s_i

	L	M	R
U	4 , 3	5 , 1	6 , 2
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all players know that all players know;
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- Mixed strategy $\sigma_i : S_i \rightarrow [0,1]$ Probability distribution over pure strategies (statistically independent for each player);
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- Thus: $\sigma_i(s_i)$ is the probability that player i assigns to strategy (action) s_i

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U	4 , 3	5 , 1	6 , 2
M	2 , 1	8 , 4	3 , 6
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Notation: Strategic Form Games

- Two Player zero sum game: $\forall s : \sum_{i=1}^{2} u_i(s) = 0$

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Sense of Mixed Strategy Concept

- Example: Rock Paper Scissors

	Rock	Paper	Scissors
Rock	0 , 0	-1 , 1	1 , -1
Paper	1 , -1	0 , 0	-1 , 1
Scissors	-1 , 1	1 , -1	0 , 0

- no pure NE, but mixed NE if both play (1/3, 1/3, 1/3)

Notation: Strategic Form Games

- Space of mixed strategies for player i: Σ_i

- Space of mixed strategy profiles: $\Sigma = x_i \Sigma_i$

- Mixed strategy profile $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_I) \in \Sigma$

- Player i's payoff when a mixed strategy profile σ is played is

$$\sum_{s \in S} \left(\prod_{j=1}^{I} \sigma_j(s_j) \right) u_i(s)$$

denoted as $u_i(\sigma)$, is a linear function of the σ_i

- A pure strategy of a player is a special mixed strategy of that player with one probability equal to 1 and all others equal to 0

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Let

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$\sigma_2(L)=0$, $\sigma_2(M)=1/2$, $\sigma_2(R)=1/2$

or short

$\sigma_1 = (1/3, 1/3, 1/3)$
$\sigma_2 = (0, 1/2, 1/2)$

We then have:

$u_1(\sigma_1, \sigma_2) = 1/3\ (0*4 + ½*5 + ½*6)$
$+ 1/3\ (0*2 + ½*8 + ½*3) +$
$1/3\ (0*3 + ½*9 + ½*2) = 11/2$

$u_2(\sigma_1, \sigma_2) = \ldots = 27/6$

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Games in Strategic Form & Nash Equilibrium

- What is rational to do?
- No matter what player 1 does: R gives player 2 a strictly higher payoff than M. „M is strictly dominated by R“
- → player 1 knows that player 2 will not play M → U is better than M or D
- → player 2 knows that player 1 knows that player 2 will not play M → player 2 knows that player 1 will play U → player 2 will play L
- This elimination process: „iterated strict dominance“

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- New example:
- Player 1: M not dominated by U and M not dominated by D
- But: If Player 1 plays $\sigma_1 = (1/2, 0, 1/2)$ he will get $u(\sigma_1) = 1/2$ regardless how player 2 plays
- → a pure strategy may be dominated by a mixed strategy even if it is not strictly dominated by any pure strategy

	L	R
U	2, 0	-1, 0
M	0, 0	0, 0
D	-1, 0	2, 0

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More Notation:

- Discussing player i's strategy-options, holding other player's options fixed:
- $s_{-i} \in S_{-i}$: „other player's strategies“
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- If $u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$ we also have $u_i(\sigma'_i, \sigma_{-i}) > u_i(s_i, \sigma_{-i})$ for all $\sigma_{-i} \in S_{-i}$ because $u_i(\sigma'_i, \sigma_{-i})$ is a convex function of $u_i(\sigma'_i, s_{-i}), u_i(\sigma'_i, s'_{-i}), u_i(\sigma'_i, s''_{-i}), \ldots$

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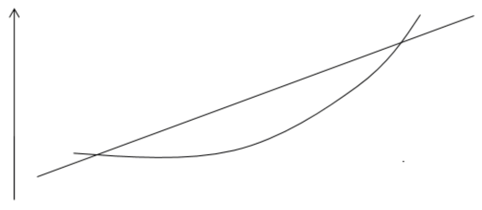
Strictly Convex function:

$f(tx+(1-t)y) < tf(x) + (1-t)f(y)$



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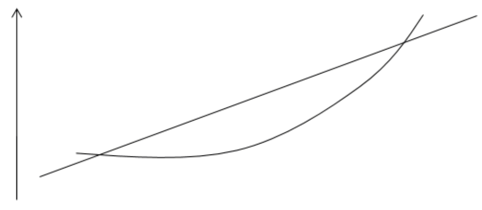

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