

Title: groh: profile1 (10.06.2015)

Date: Wed Jun 10 08:17:02 CEST 2015

Duration: 79:14 min

Pages: 64

PD Dr. Georg Groh

Homophily and Distance

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 - data (e.g. filenames) („keys“) and host-IDs (e.g. IP-addresses) („nodes“) hashed into the same m-dim key-space,
 - Key k is assigned to node $successor(k)$,
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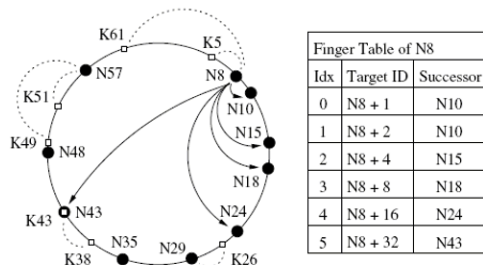


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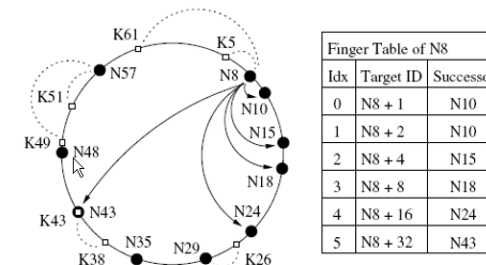


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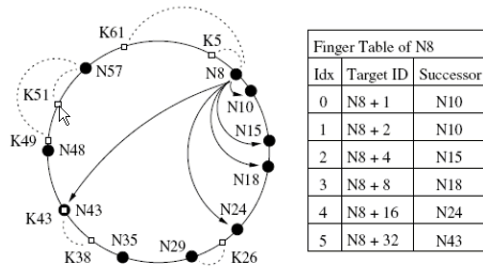


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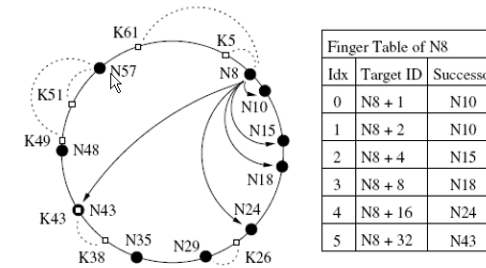


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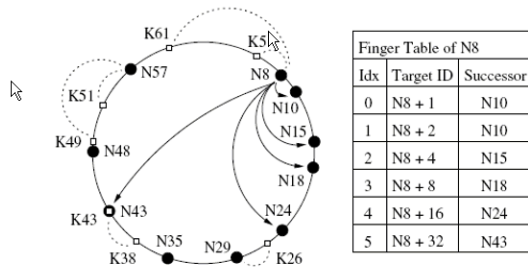


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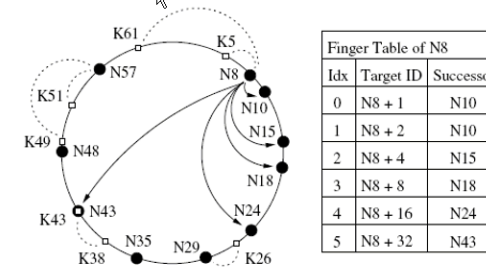


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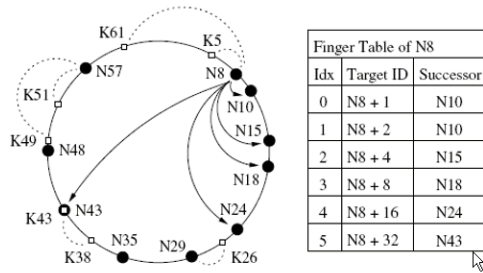


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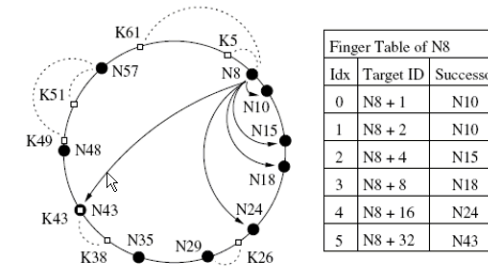


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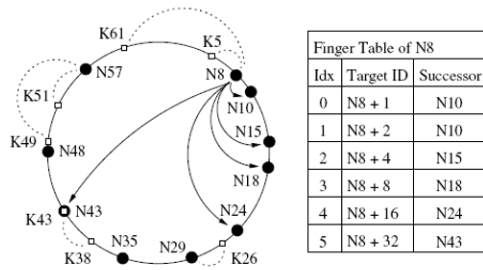


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Kleinberg Model

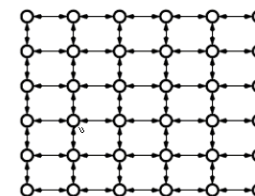
- Put nodes on a $n \times n$ grid. **Distance**: Manhattan:

$$r(i,j) = |x_i - x_j| + |y_i - y_j|$$
- Each node i : Connected to all nodes with $r(i,j) \leq q_1$ (regular **local contacts**)
- Each node i : **Additional q_2 other „long range“ edges**:
Probability of edge to node j :

$$P(j) \sim r(i,j)^{-\alpha}$$

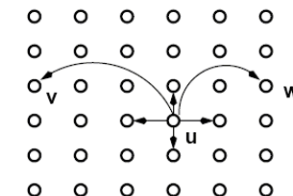
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$q_1 = 1, q_2 = 0$

B)



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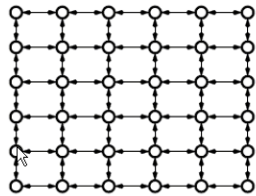
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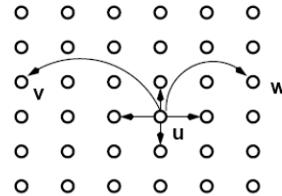
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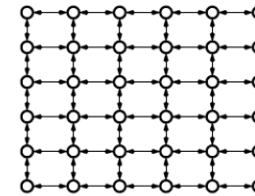
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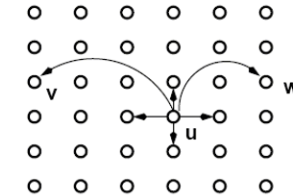
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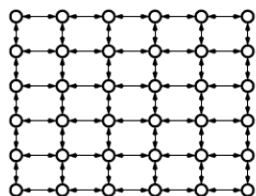
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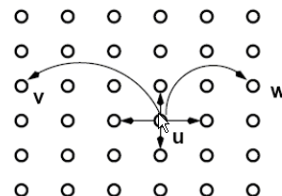
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Kleinberg Model

- local (decentralized) knowledge:**

- Each node only knows only:

- Its adjacent nodes
- The grid's principle structure
- Position of target node on the grid
- Positions and long-range contacts of nodes on the message path so far

- (Search-) algorithm with only local knowledge: „decentralized“





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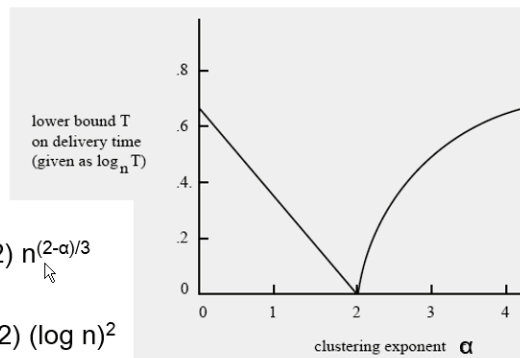


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- Now: Send message with local (decentralized) knowledge only

- Given: Decentralized greedy message delivery algorithm: measure number of expected delivery steps s :

- $0 \leq \alpha < 2$: s at least
 $\sim c_1(\alpha, q_1, q_2) n^{(2-\alpha)/3}$
- $\alpha = 2$: s at most
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(closely after [3])

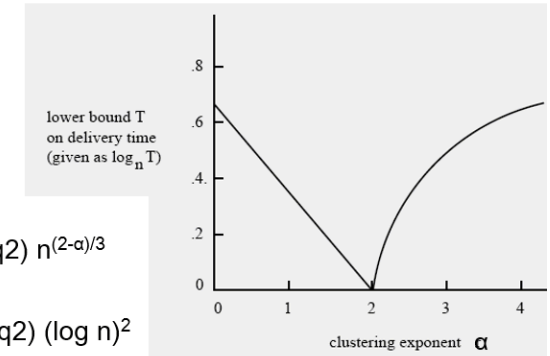


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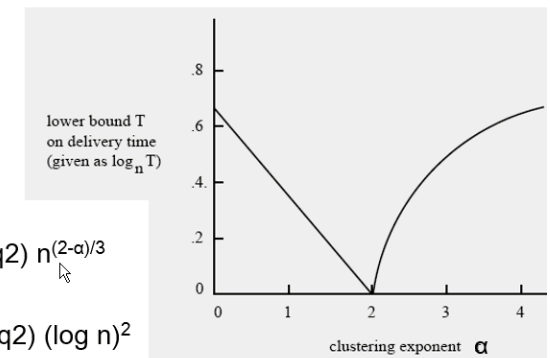


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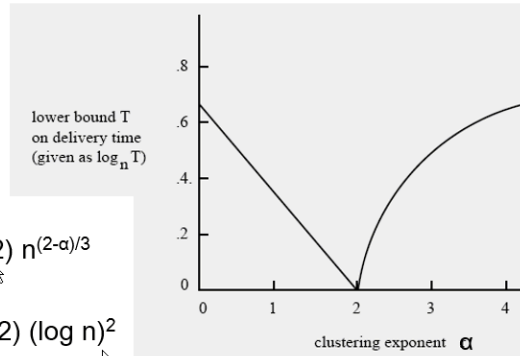


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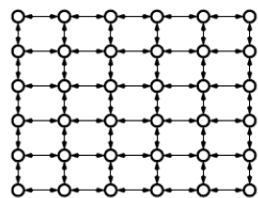
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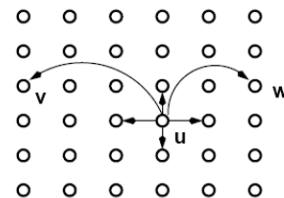
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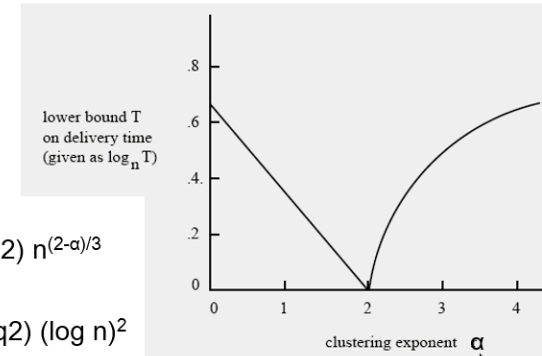


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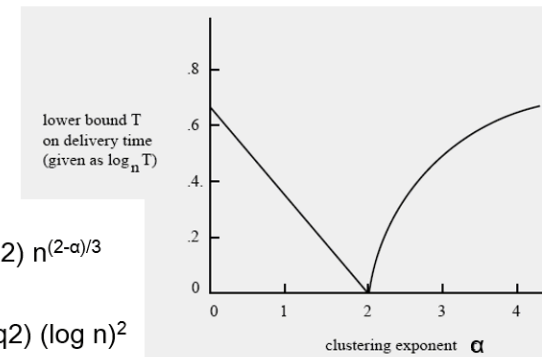


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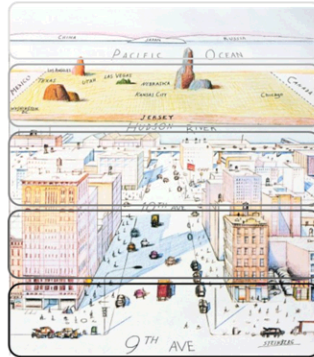


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- Effectively: Only a notion of distance (not necess. spatial!) is necessary to route!
- Applications in P2P Systems (see [2])

Explanation for the dependency of $\alpha=D$ on the grid's dimension D for efficient delivery:

- start from node u
- partition the set of other nodes into sets $A_0, A_1, A_2, \dots, A_{\log n}$, where A_i has a distance to u between 2^i and 2^{i+1}
- proven in [3]: only $\alpha=D$ ensures that the q^2 „long-range“ contacts are evenly distributed over the A_i
- for $\alpha>D$: bias towards smaller distances;
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[5,6]

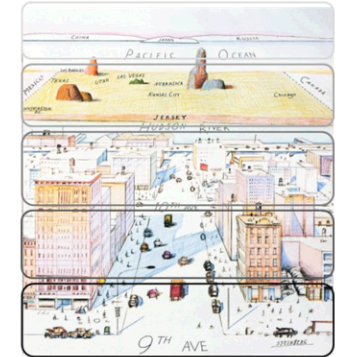


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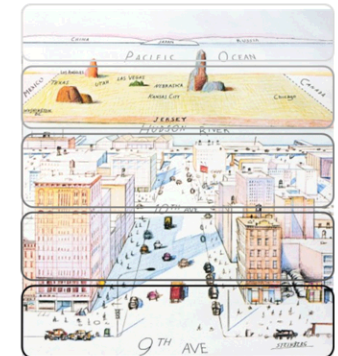


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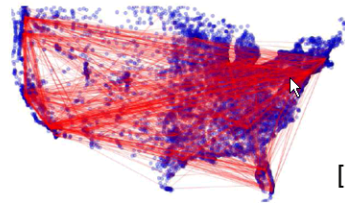
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Geographic Distance as Routing Metric

- In **analysis of Milgram's experiment**:
 - People **early in the resp. path**: often cite **geographic proximity** as main forwarding criterion;
 - People **late in the path**: chose **similarity of occupation**
- [6]: Study „Kleinberg-like“ distributions / effects on **real social NW**:
 - LifeJournal.com** : locate $\sim 10^6$ users (long/lat of their hometown)
 - Simulate Milgram** on friendship NW with greedy decentralized forwarding and geographical proximity as criterion
 - Result: **efficient routing is possible** on average (see [6])



[7]

Fig. 2 The LiveJournal social network [32]. A dot is shown for each geographic location that was declared as the hometown of at least one of the $\approx 500,000$ LiveJournal users whom we were able to locate at a longitude and latitude in the continental United States. A random 0.1% of the friendships in the network are overlaid on these locations.



Geographic Distance as Routing Metric

- But: Investigate Kleinberg's $\alpha = 2$ claim: result: **probability of friendship as a function of distance** reveals $\alpha \approx 1$

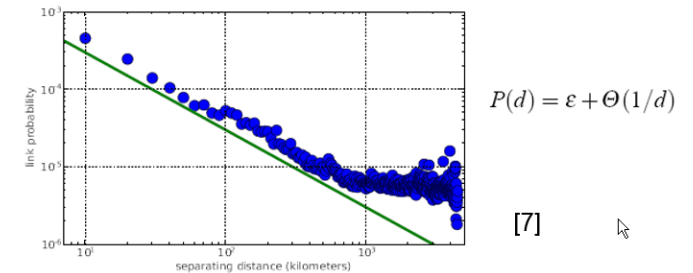


Fig. 3 The probability $P(d)$ of a friendship between two people in LiveJournal as a function of the geographic distance d between their declared hometowns [32]. Distances are rounded into 10-kilometer buckets. The solid line corresponds to $P(d) \propto 1/d$. Note that Theorem 1 requires $P(d) \propto 1/d^2$ for a network of people arranged in a regular 2-dimensional grid to be navigable.

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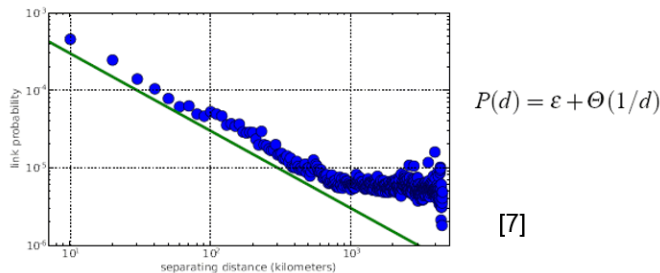


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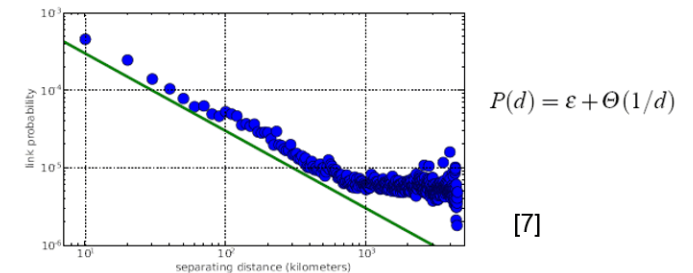


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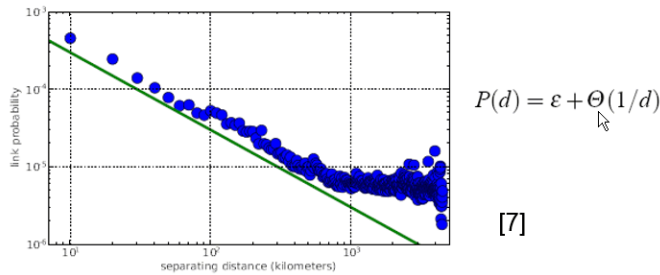


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Geographic Distance as Routing Metric

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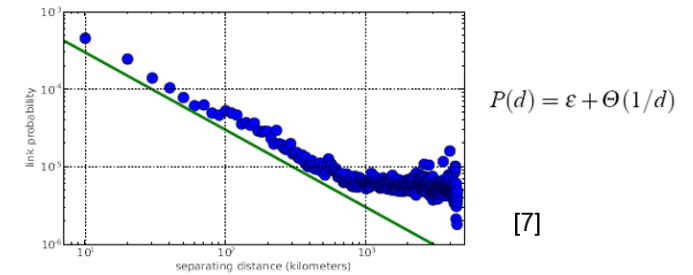


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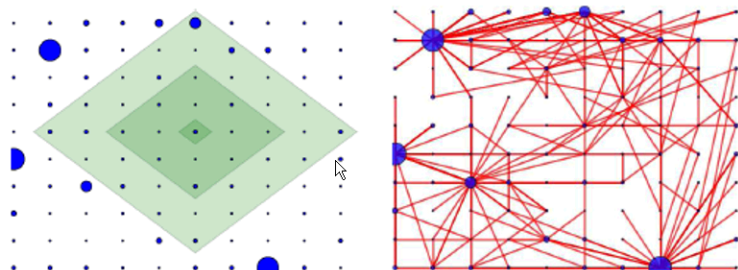
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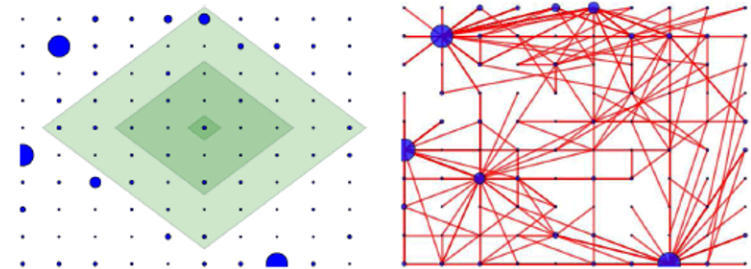


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\rightarrow In Kleinberg’s version each node is efficiently reachable; Here: node-pairs are chosen uniformly randomly \rightarrow connections from dense grid points to other dense grid points are more likely. However: here: persons in thinly populated areas might be hard to route to.



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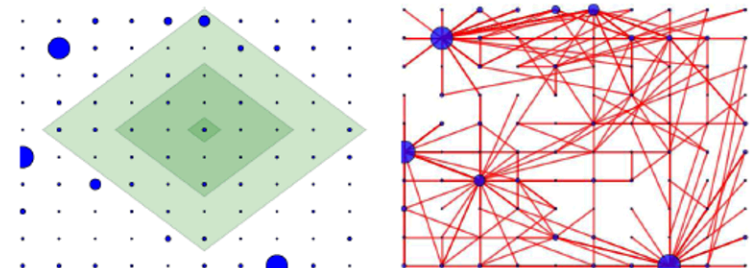
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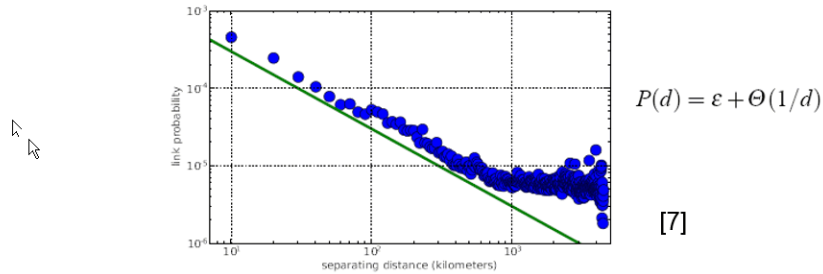
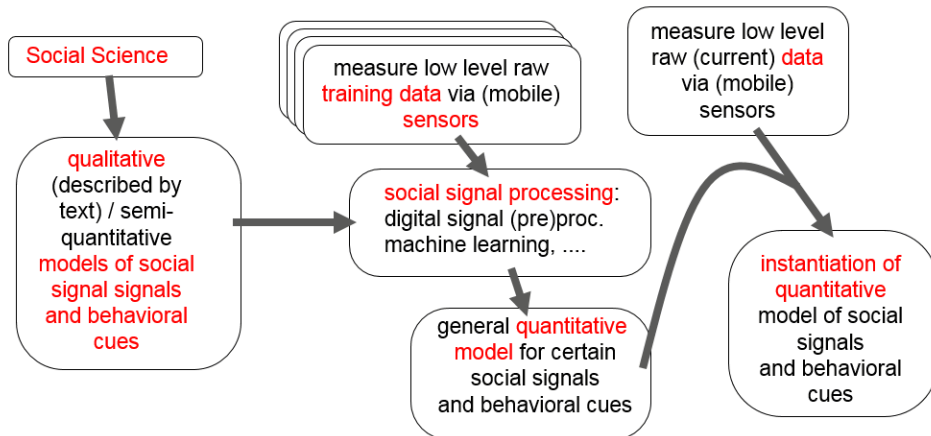


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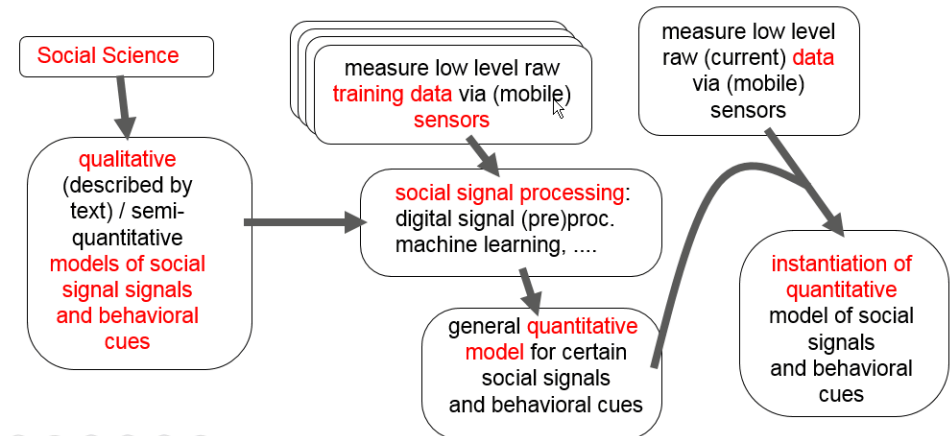
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- General field: Human behavior modeling
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 - individual behavior models often need (lots of) individual training data
- Basic social behavior
 - follows strict, nearly uniform rules (within certain cultural frames (groups))
 - social behavior models can exploit these uniform rules
 - models may not have to be specially trained for individuals

