Script **generated by TTT**

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Pages: 87

Shortest Paths: Shortest Path Betweenness

- Again assume that communication (workflows etc.) happen along shortest paths only. Let

$$\delta_{ab}(v) = \frac{\sigma_{ab}(v)}{\sigma_{ab}}$$

with σ_{ab}: total number of shortest paths between nodes a and b.

Interpretation. Probability that v is involved in a communication between a and b

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with σ_{ab}: tot nu r of sho st pa nodes a and b. G =(V,E) t n u ipl. G' = ed nal e es as s. T G' gina edges are connected G na in tl al node

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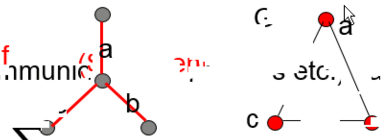
Shortest Paths: Shortest Path Betwee centra



- Again assume Be that communi o f ent s etc., along shortest pPB is r '. Le s appear an pea $v \in V$ vertex stress centrali for node x. Nu ber of shortest e x; Straightforward vers on f he e: umbe of shortest

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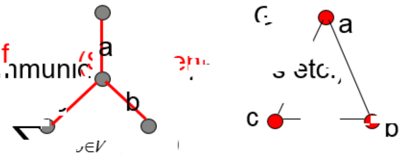

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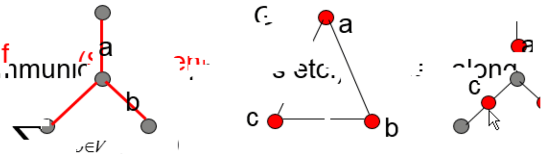

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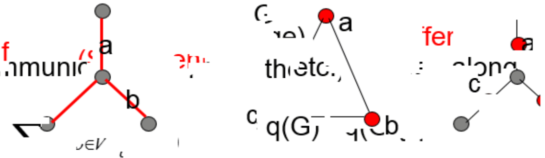
Shortest Paths: Shortest Path Betwee centra ...



- Again assume Be that co..munic o f ... shortest pPB is r ... Le ... s appear ... ian pea ... thetc., ... q(G) ... fer ... along

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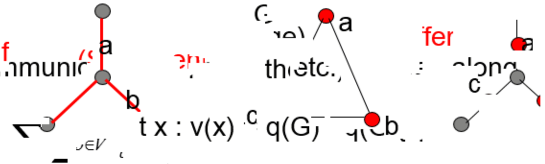

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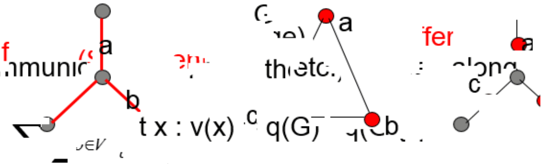

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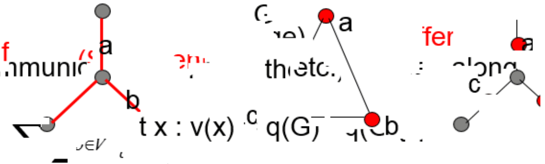

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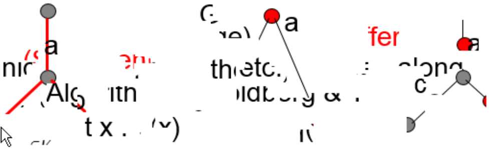

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ce ... f respect capacity contraints: $0 \le \tilde{f}(e) \le c(e)$

vertex ... at r ... es:

$$\forall v \in V\setminus\{s,t\}: \sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$$

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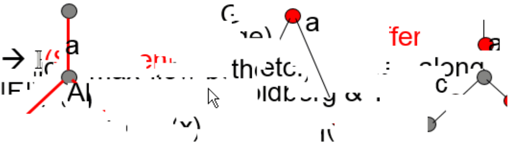

- Again computing a flo f: E → ... shortest : O(|V| |E| log(|V|²/|E|)) Al

vertex stress $\frac{f_{st}(G) - f_{st}(G\setminus\{u\})}{f_{st}(G)}$ ... Number of shortest ... x; Str ... e: number of shortest

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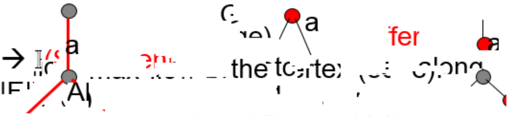

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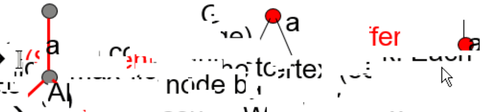

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Shortest Paths: Shortest Path Retwee ceatra

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$\forall v \in V \setminus \{s,t\}$: $\sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$

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Shortest Paths: Shortest Path Retwee ceatra

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Shortest Paths: Shortest Path Retwee ceatra

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Shortest Paths: Shortest Path Betwee centra

- Again input … flow f: E → … shortest … (stress … E| log(|V … tion … tance … node b … w

$$\sum_{a\in V;a\neq v}\sum_{b\in V;b\neq v}\sigma_{ab}(v) \quad f(\bar{v}) \quad \sum_{w\in V} \quad \sum_{t\in V}$$

with σ … tot … $q(G) = \sum_{s,t\in} \frac{1}{2}$ … the … er of … test p … dut in

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a and b … $(s,t, …$ … vitality" of graph … elc … litie

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$$\forall v \in \ldots s,t\}: \quad \sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$$

Shortest Paths: Shortest Path Betwee centra

- Again input … flo … f: E → … shorte … mple … og(|V … tion … tance … node b … w

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$$\sum \sum f(\bar{v}) \quad \sigma_{ab} \quad \sum_{w\in V} \quad \sum_{t\in V}$$

with σ … tot … $q(G) = \sum_{s,t\in} \frac{1}{2}$ … the … er of … dut in

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- Again input … flo … f: E → … shorte … mple … og(|V … tion … likaners … node b … w ha … n't … l th … ir l

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σ_{cd}=1 (leng … 3) … f … k … $c(x) = q(G) - q(\ldots)$

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$$\forall v \in \ldots s,t\}: \quad \sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$$

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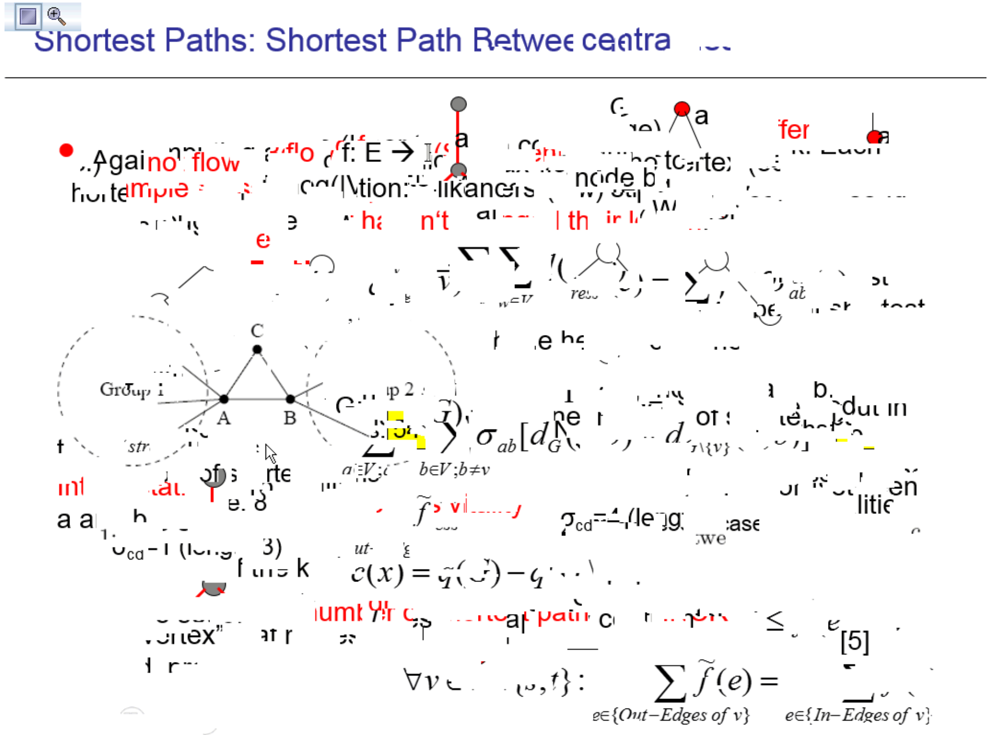

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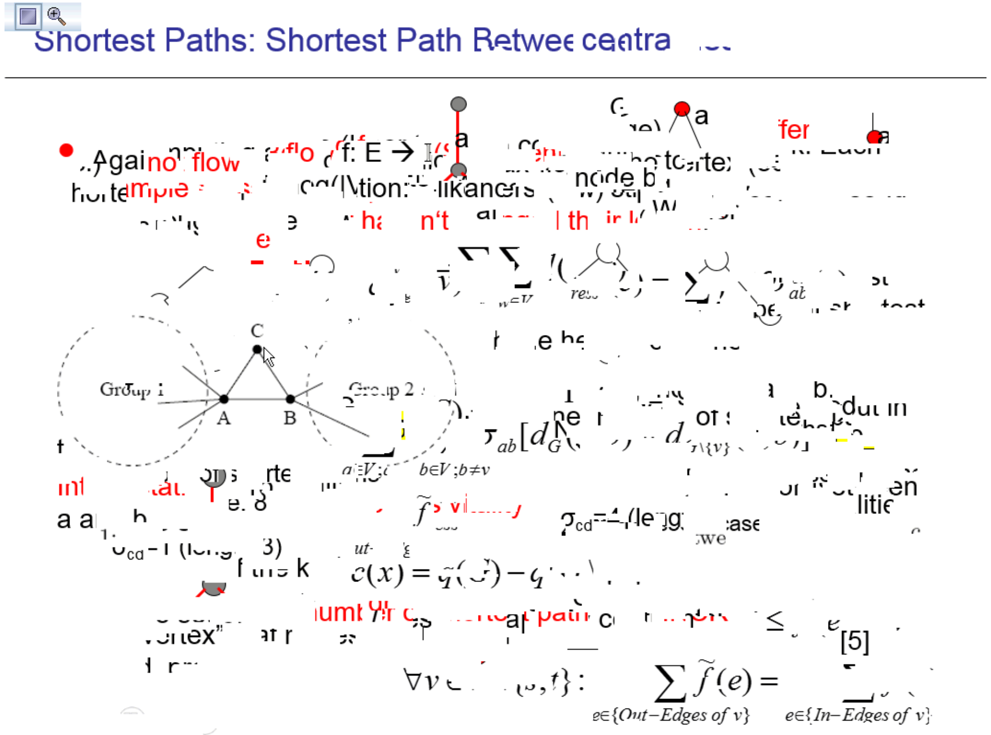

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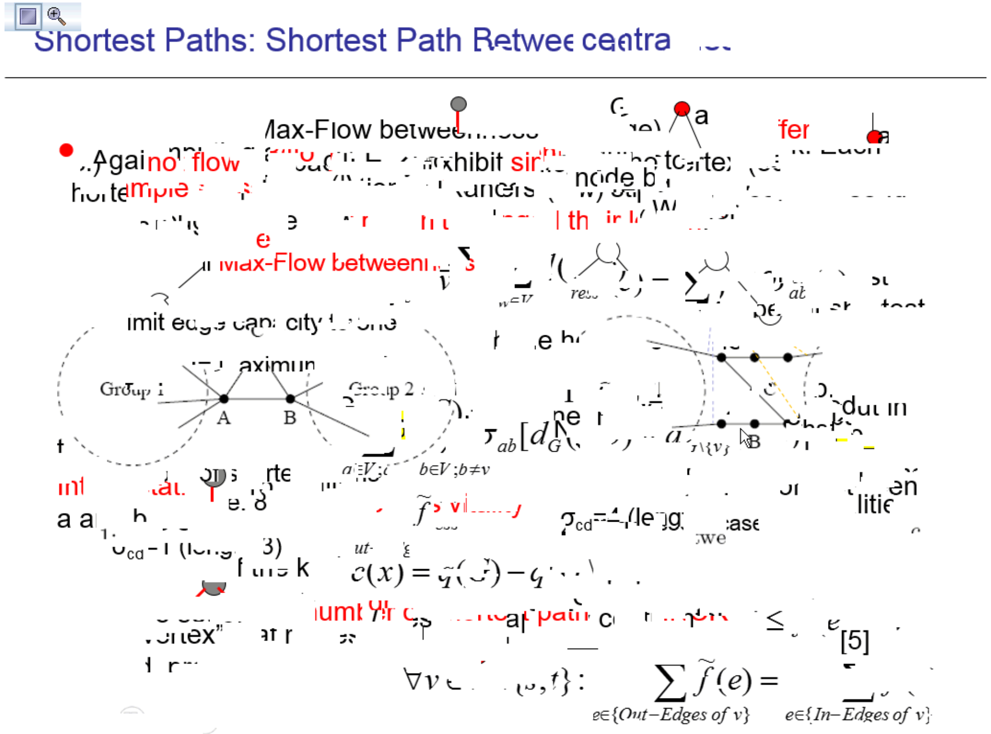

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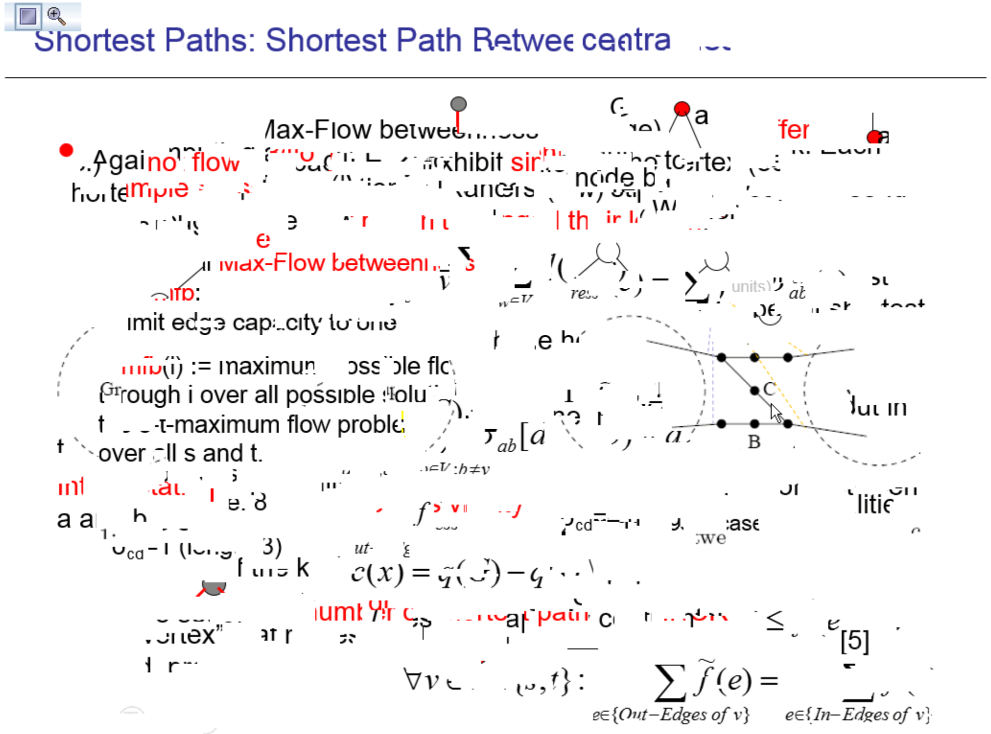
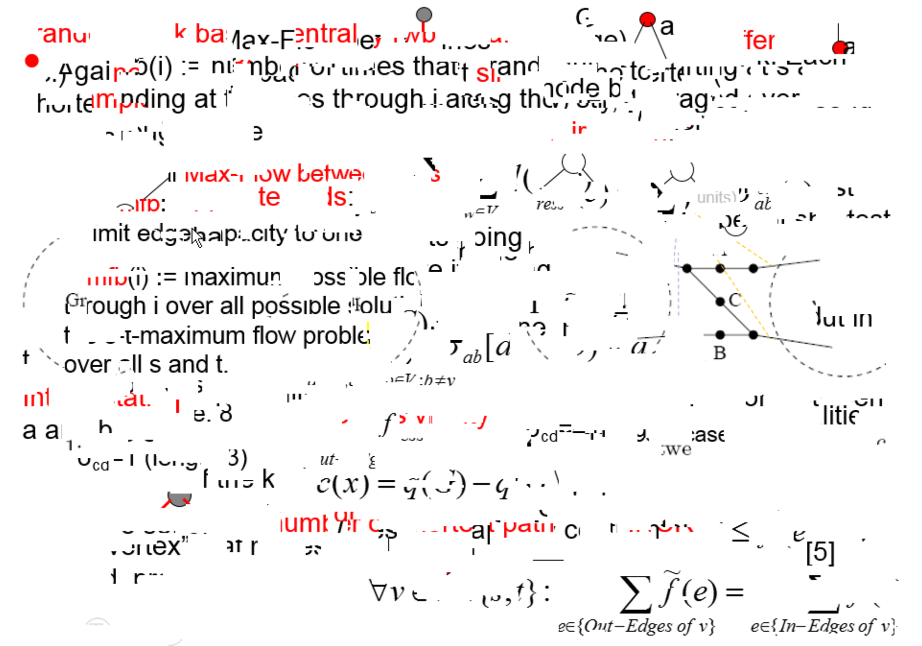


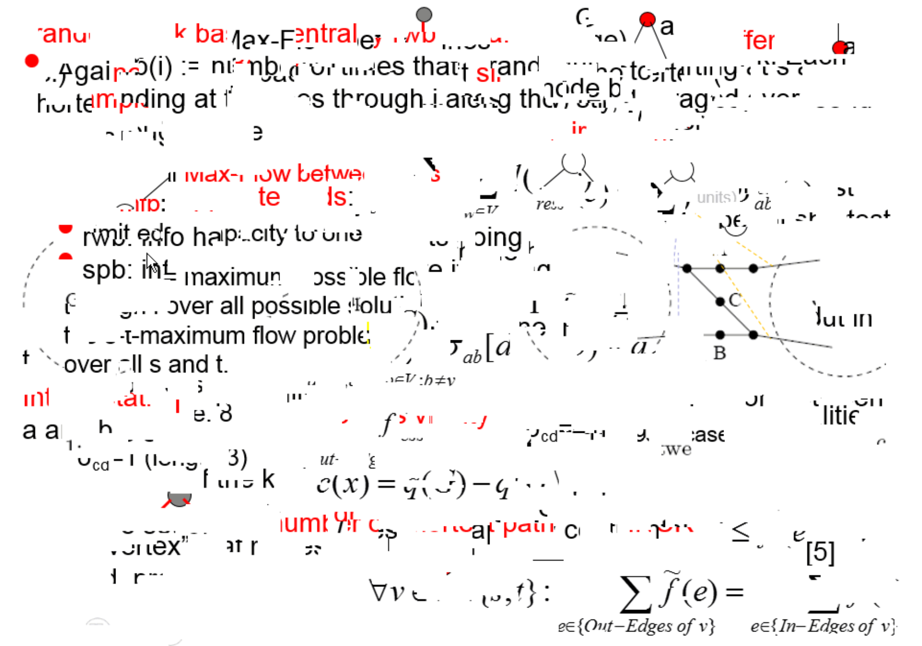
Shortest Paths: Shortest Path Retwee centra



over all s and t.

$$\forall v \ldots : \sum_{e \in \{Out-Edges\ of\ v\}} \widetilde{f}(e) =$$

Shortest Paths: Shortest Path Retwee centra



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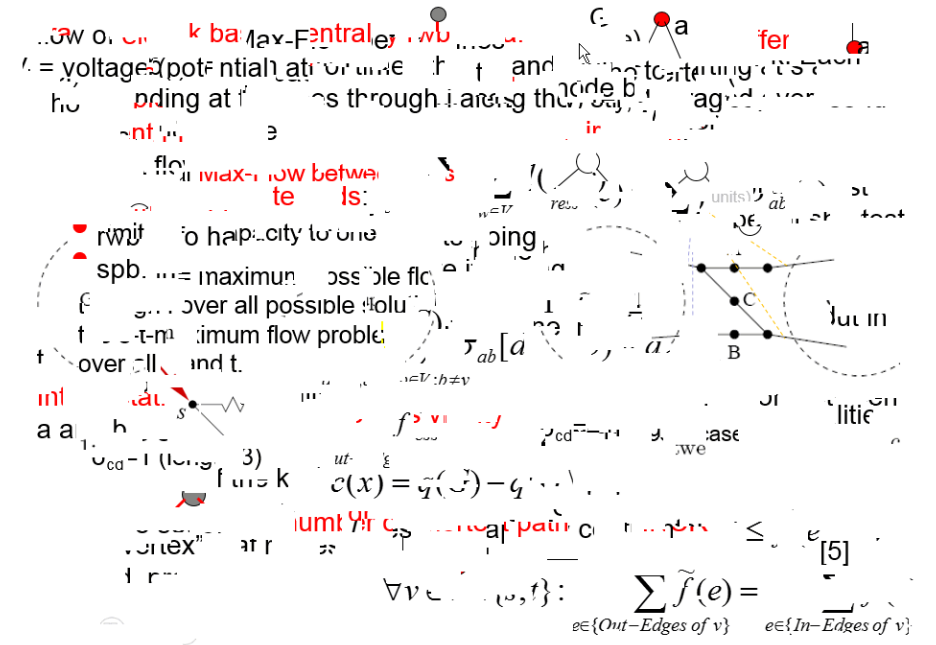
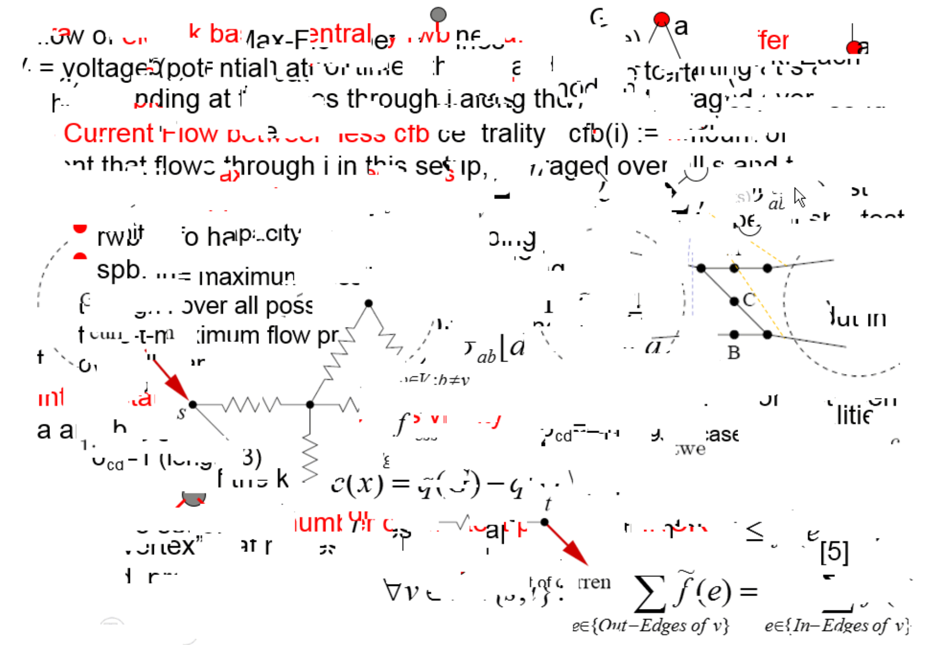


= voltage (potential at

$$\forall v \ldots : \sum_{e \in \{Out-Edges\ of\ v\}} \widetilde{f}(e) =$$

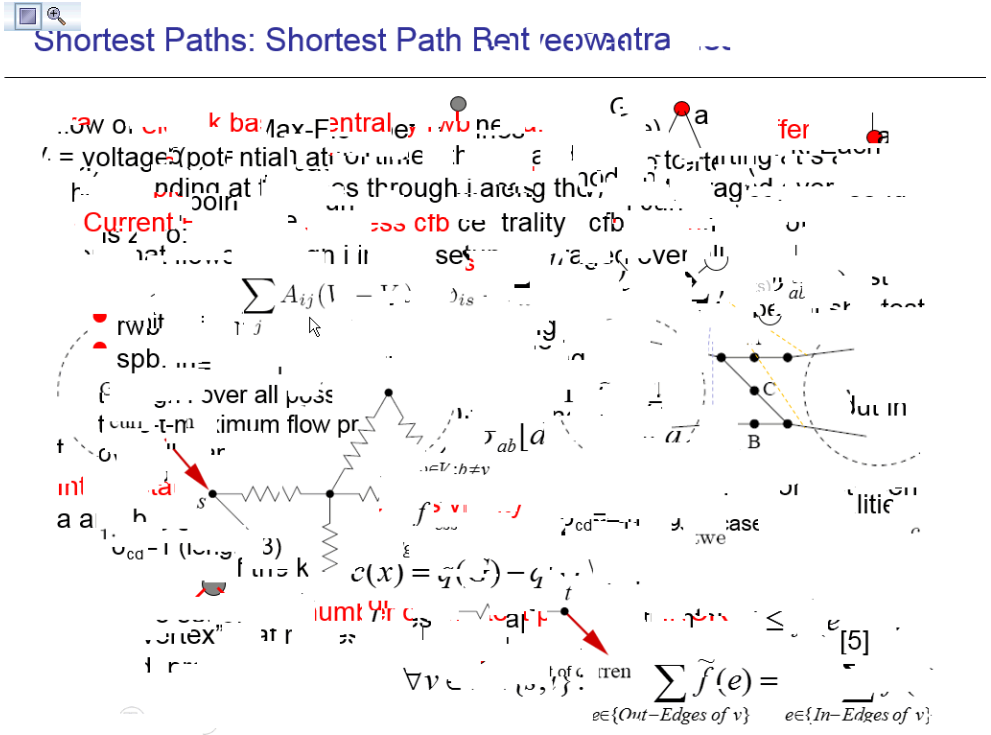
Shortest Paths: Shortest Path Rent 'eeWeatra



= voltage (potential at

Current Flow ... ctb ce trality cfb(i) := ...

$$\forall v \ldots : \sum_{e \in \{Out-Edges\ of\ v\}} \widetilde{f}(e) =$$



Shortest Paths: Shortest Path Rent 'eewaatra

Max-F entral

= voltage (potential at

pding at

s through

Current

is 2

$$\sum_{j} A_{ij}(V_i - V_j) = \delta_{is} - \delta_{it},$$

rw

spb.

over all poss

imum flow pr

$\sigma_{ab}[a$

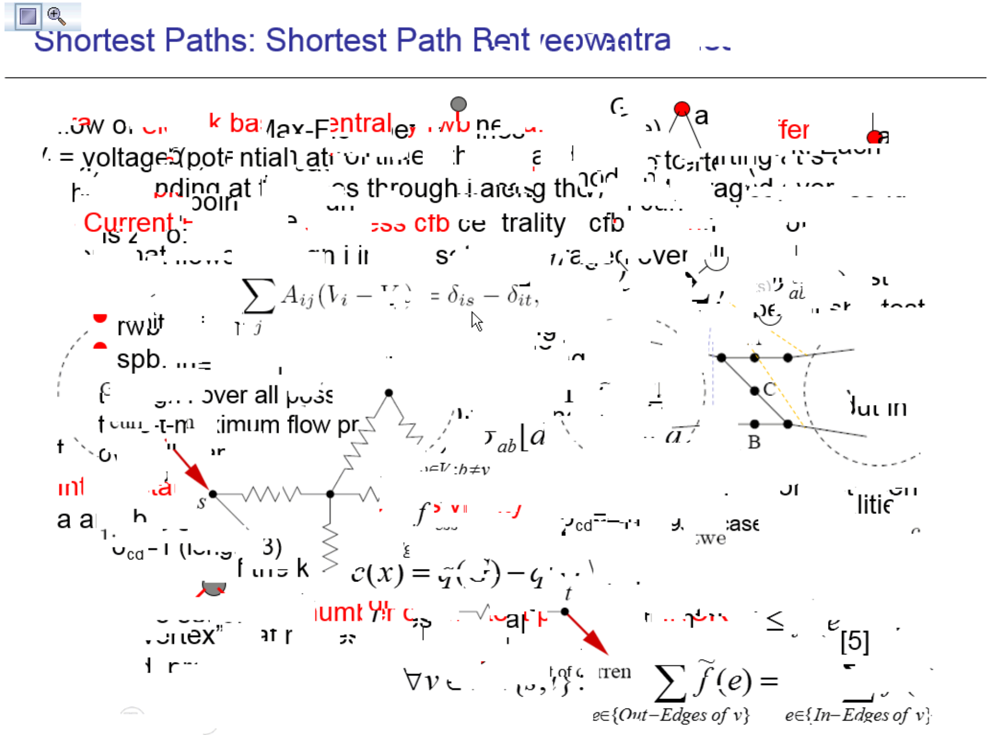
$c(x) = q(\cdot) - q$

imum

vertex"

[5]

$$\forall v \in V \setminus \{s,t\}: \sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$$



Shortest Paths: Shortest Path Rent 'eewaatra

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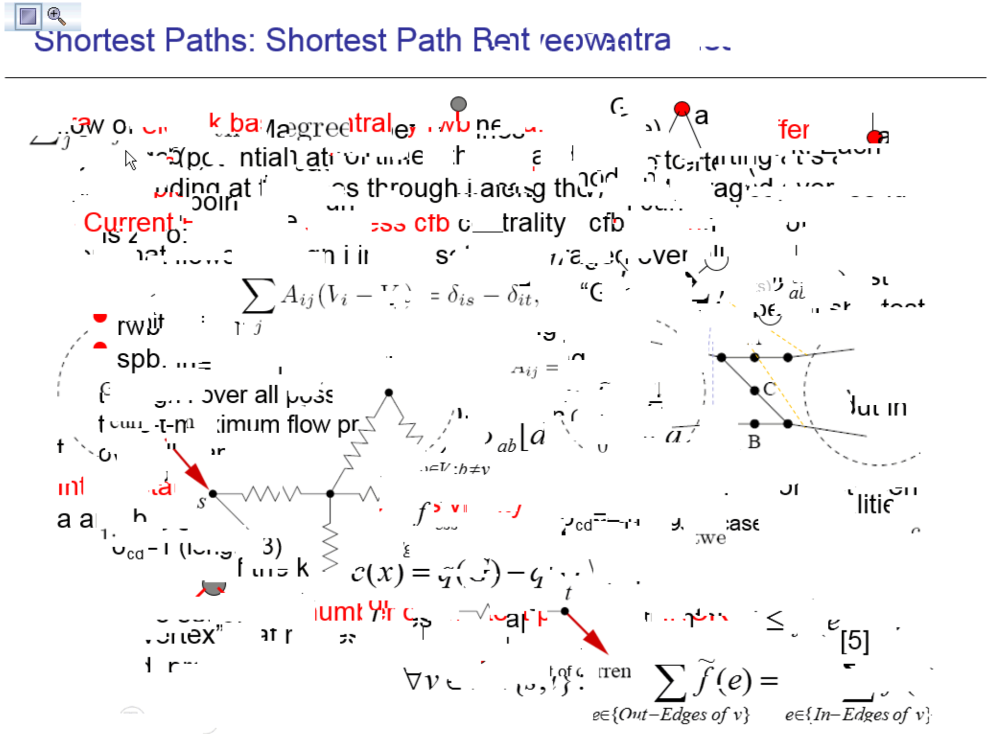

Shortest Paths: Shortest Path Rent 'eewaatra

degree

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Current

$$\sum_{j} A_{ij}(V_i - V_j) = \delta_{is} - \delta_{it},$$

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over all poss

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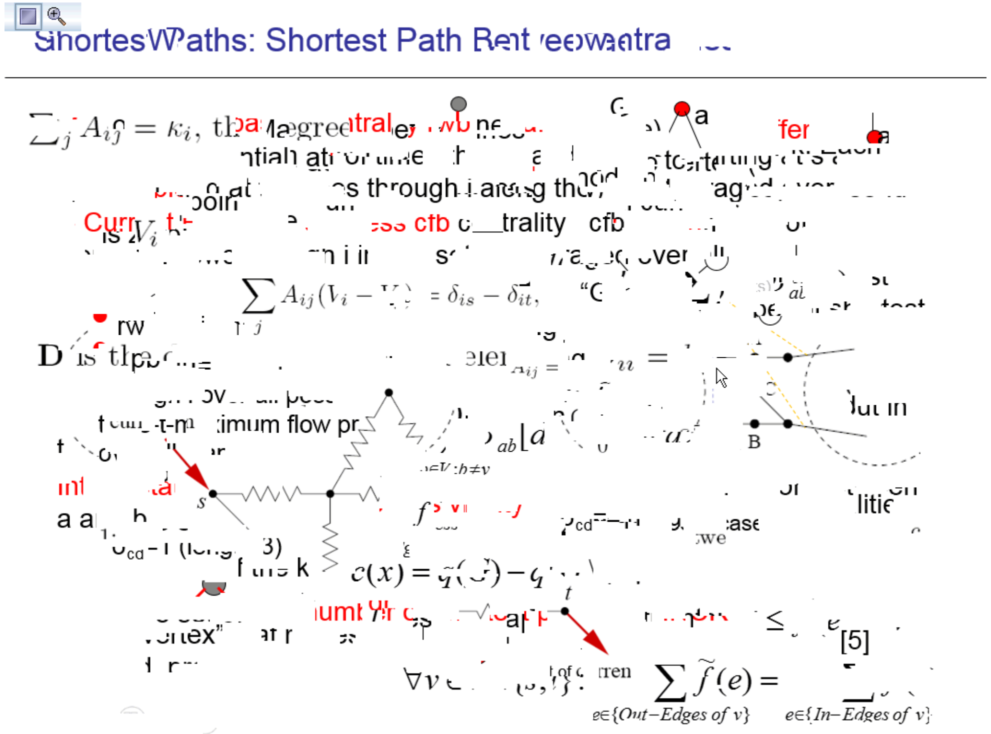

Shortest Paths: Shortest Path Rent 'eewaatra

$\sum_j A_{ij} = k_i$, the degree

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V_i

$$\sum_{j} A_{ij}(V_i - V_j) = \delta_{is} - \delta_{it},$$

D is the

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$\sigma_{ab}[a$

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$$\forall v \in V \setminus \{s,t\}: \sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$$

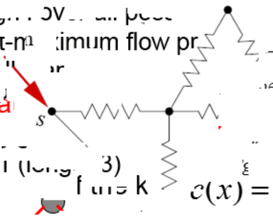
ShortesWPaths: Shortest Path Rent 'eowaatra

$\sum_j A_{ij} = k_i$, the degree ... ntia ... rough ... th

$$\sum_j A_{ij}(V_i - V_j) = \delta_{is} - \delta_{it} \xrightarrow{} \underbrace{(\mathbf{D}-\mathbf{A})}_{\text{“Graph Laplacian”}} \cdot \mathbf{V} = \mathbf{s}$$

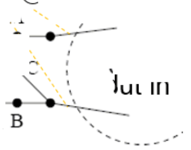
$= \delta_{is} - \delta_{it}$,

$\mathbf{D}$ is the ... $A_{ij} =$ ... $= k_i$



... maximum flow pr... a ... s, for ... t, 0 otherwise.

... litie ... ase



$c(x) = q(x) - q$ ... [5]

... vertex ...

$\forall v$ ... $\sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$

ShortesWPaths: Shortest Path Rent 'eowaatra

$\sum_j A_{ij} = k_i$, the degree ... ntia ... rough ... th

$$\sum_j A_{ij}(V_i - V_j) = \delta_{is} - \delta_{it} \xrightarrow{} \underbrace{(\mathbf{D}-\mathbf{A})}_{\text{“Graph Laplacian”}} \cdot \mathbf{V} = \mathbf{s}$$

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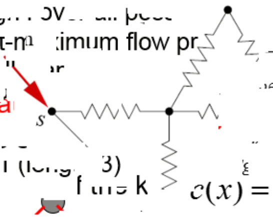
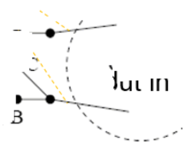


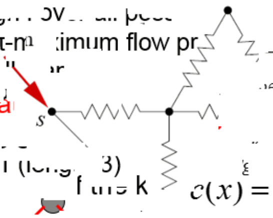


ShortesWPaths: Shortest Path Rent 'eowaatra

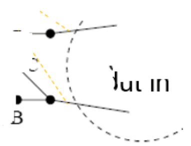
$\sum_j A_{ij} = k_i$, the degree ... ntia

Laplacian is not invertible, ... because ... determined → set one $V_v = 0$ and measure voltage relati... and column ... since ... “Graph Laplacian”

$\mathbf{D}$ is the ... $A_{ij} =$ ... $= k_i$



... maximum flow pr... a ... s, for ... t, 0 otherwise.



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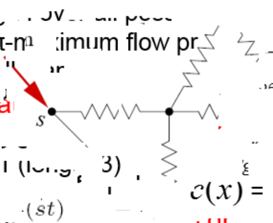
$\forall v$ ... $\sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$

ShortesWPaths: Shortest Path Rent 'eowaatra

$\sum_j A_{ij} = k_i$, the degree ... ntia

Laplacian is not invertible, ... because ... determined → set one $V_v = 0$ and measure voltage relati... remove the v-th row and column, since $V_v = 0$ → ... invertible

$(\mathbf{D}_v - \mathbf{A}_v)$ ... $= k_i$



... maximum flow pr... a ... back into $(\mathbf{D}_v - \mathbf{A}_v)$ ... the $\mathbf{T}$. ... otherwise.

$V_i^{(st)}$ ... $c(x) = q(x) - q$ ... δ_{is} ... [5]

$I_i^{(st)}$ ... $\sum_{e\in\{Out-Edges\ of\ v\}} \tilde{f}(e) = \sum_{e\in\{In-Edges\ of\ v\}}$

Shortes Paths: Shortest Path Ret eewaatra

$\sum_j A_{ij} = k_i$, th

$\sum_j A$ erdet mined → set one $V_v = 0$ and

remove the v-th row and column

$(\mathbf{D}_v - \mathbf{A}_v)$

imum flow pr

ack into $(\mathbf{D}_v - \mathbf{A}_v)$

wise.

e $\mathbf{T}$.

$V_i^{(st)}$ T_{is} T_{it}

[5]

i: $I_i^{(st)} = \frac{1}{2}\sum_j A_{ij}|V_i^{(st)} - V_j^{(st)}|$

es of v}

Shortes Paths: Shortest Path Ret eewaatra

$\sum_j A_{ij} = k_i$, th

$I_i^{(st)}$ vertible, be

$\sum_j A$ erdet set one $V_v = 0$ and

remove th $\frac{1}{2}\sum_j$ and colu δ_{it} $\neq s, t$

$(\mathbf{D}_v - \mathbf{A}_v)$

des s and t

ack into $(\mathbf{D}_v - \mathbf{A}_v)$

wise.

e $\mathbf{T}$.

$V_i^{(st)}$ T_{is} T_{it}

[5]

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des s and t

ack into $(\mathbf{D}_v - \mathbf{A}_v)$

wise.

e $\mathbf{T}$.

$V_i^{(st)}$ T_{is} T_{it}

[5]

$b_i = \frac{}{\frac{1}{2}n(n-1)}$

$= \frac{1}{2}\sum_j A_{ij}|V_i^{(st)} - V_j^{(st)}|$

es of v}

Shortes Paths: Shortest Path Ret eewaatra

$\sum_j A_{ij} = k_i$, th

$I_i^{(st)}$ around vertible, be

$\sum_j A$ erdet

net number of times through on

its journey (averaged over a large number of trials

$(\mathbf{D}_v$ s, t)

des s and

ack into $(\mathbf{D}_v - \mathbf{A}_v)$

wise.

e $\mathbf{T}$.

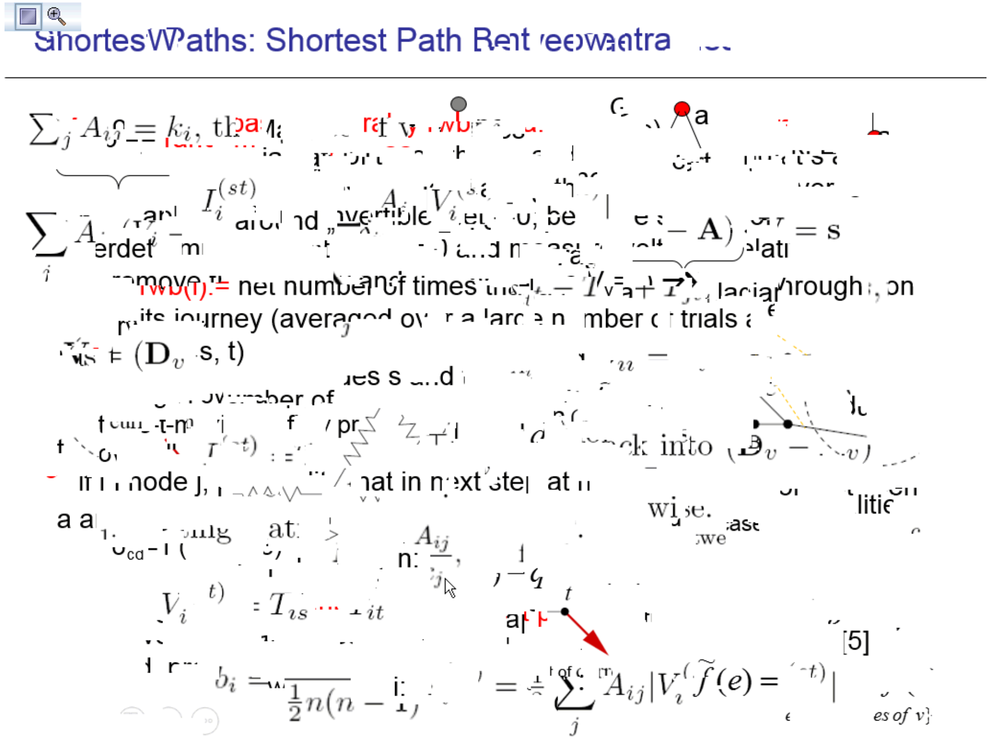
V_i T_{is} T_{it}

[5]

$b_i = \frac{}{\frac{1}{2}n(n-1)}$

$= \frac{1}{2}\sum_j A_{ij}|V_i^{(st)} - V_j^{(st)}|$

es of v}



ShortesWPaths: Shortest Path Rent 'eeweatra

$\sum_j A_{ij} = k_i$, th

$I_i^{(st)}$

net number of times ... through ... on

its journey (averaged over a large number of trials

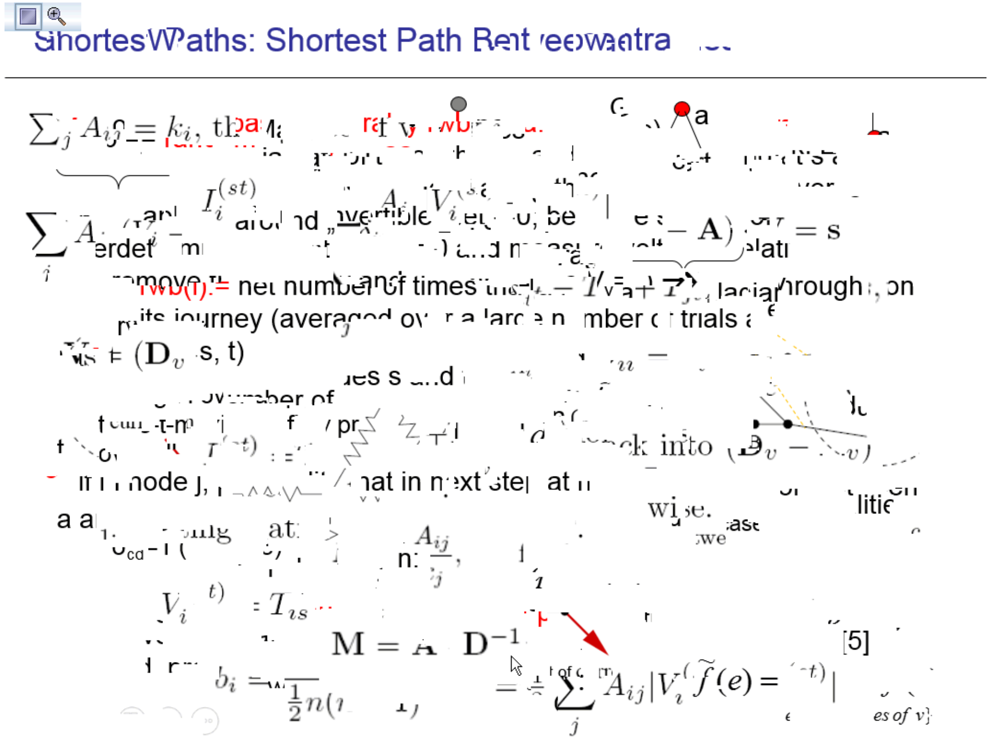
$(\mathbf{D}_v$ s, t)

node j, ... that in next step at

$b_i = \frac{1}{\frac{1}{2}n(n-1)}$

$= \sum_j A_{ij}|V_i^{(} \tilde{f}(e) =$

[5]



ShortesWPaths: Shortest Path Rent 'eeweatra

$\sum_j A_{ij} = k_i$, th

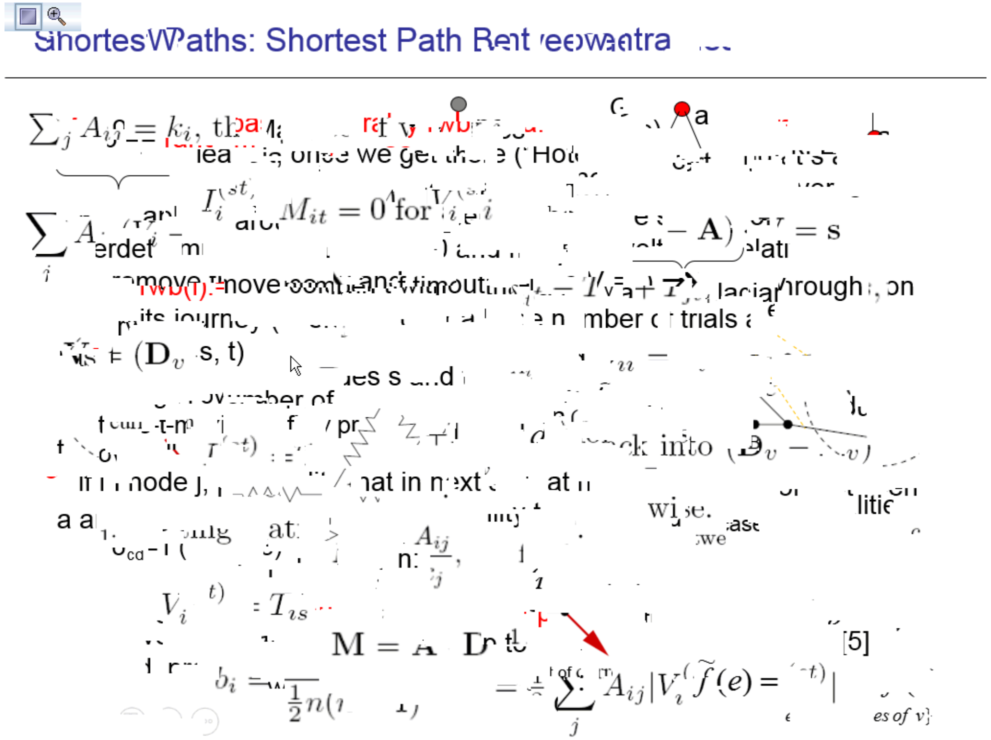
net number of times ... through ... on

its journey (averaged over a large number of trials

$\mathbf{M} = \mathbf{A} \cdot \mathbf{D}^{-1}$

$= \sum_j A_{ij}|V_i^{(} \tilde{f}(e) =$

[5]



ShortesWPaths: Shortest Path Rent 'eeweatra

$M_{it} = 0$ for $i \neq i$

remove ... without ... through ... on

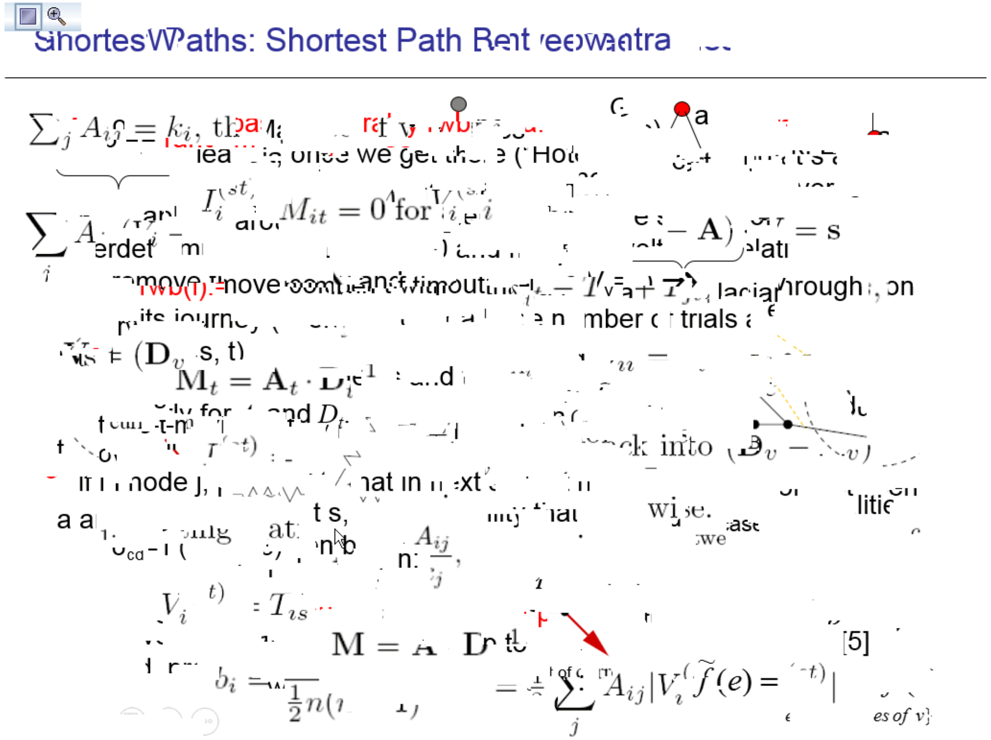
$(\mathbf{D}_v$ s, t)

node j, ... that in next ... at

$\mathbf{M} = \mathbf{A} \cdot \mathbf{D}_t$

$= \sum_j A_{ij}|V_i^{(} \tilde{f}(e) =$

[5]



ShortesWPaths: Shortest Path Rent 'eeweatra

$M_{it} = 0$ for $i \neq i$

$\mathbf{M}_t = \mathbf{A}_t \cdot \mathbf{D}_t^{-1}$

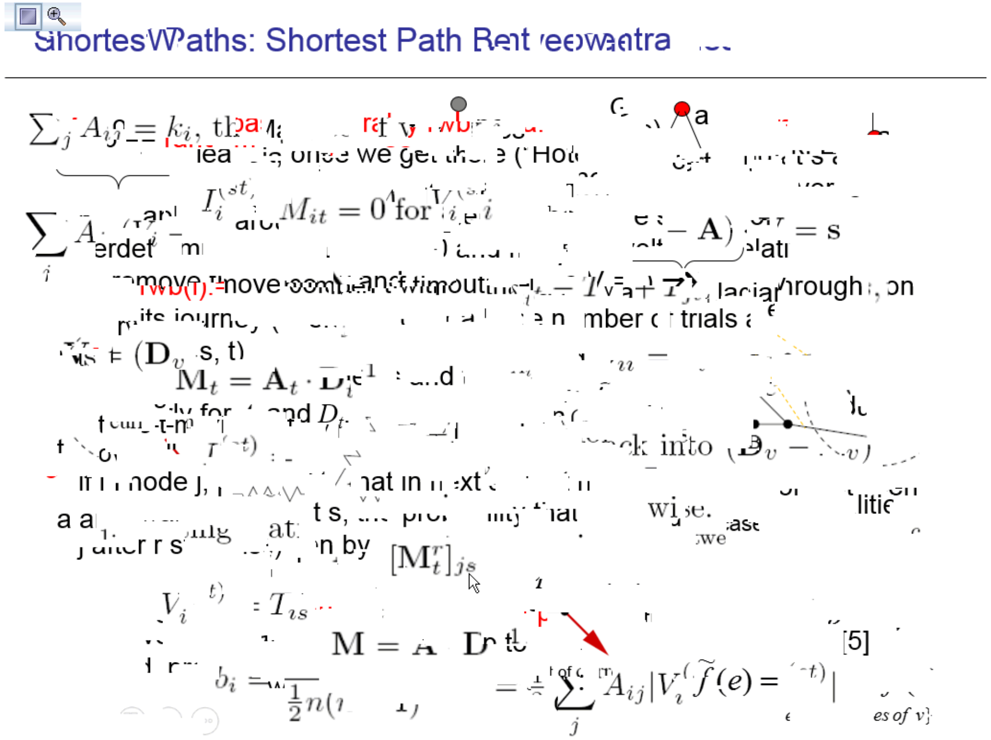
only for ... and D_t

node j, ... that in next ... at

$\mathbf{M} = \mathbf{A} \cdot \mathbf{D}_t$

$= \sum_j A_{ij}|V_i^{(} \tilde{f}(e) =$

[5]



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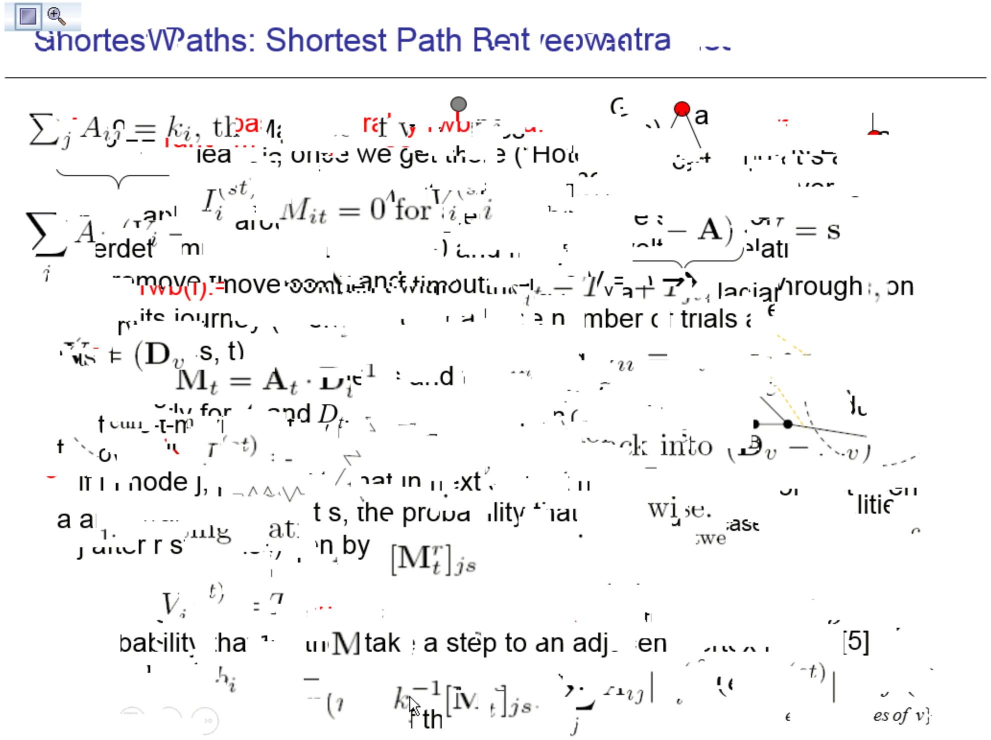

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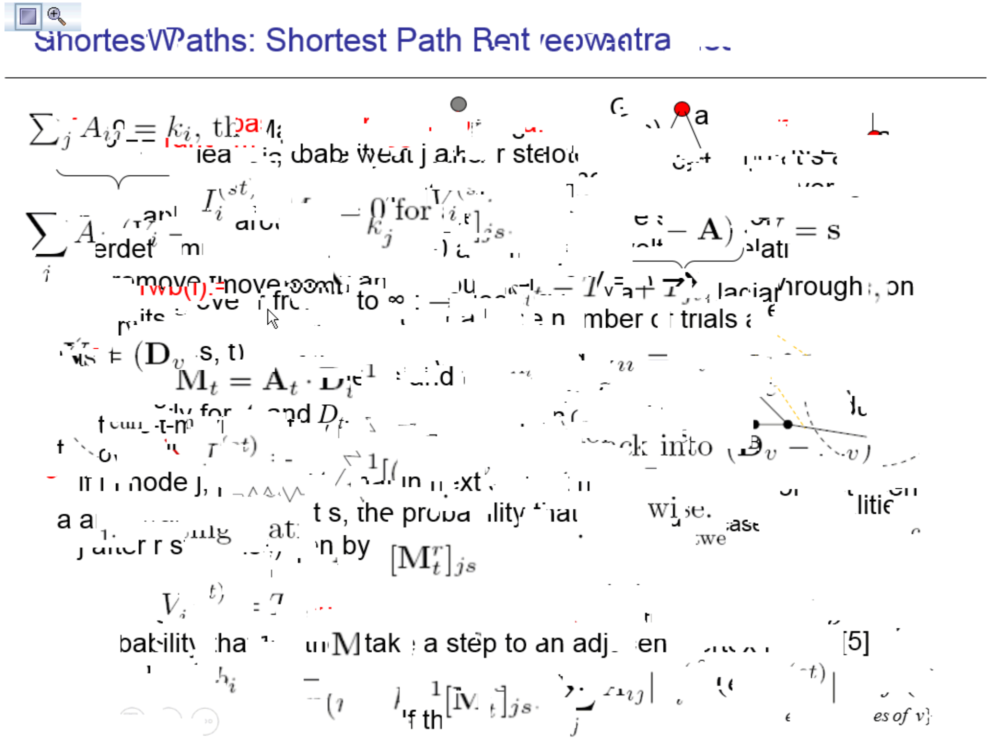

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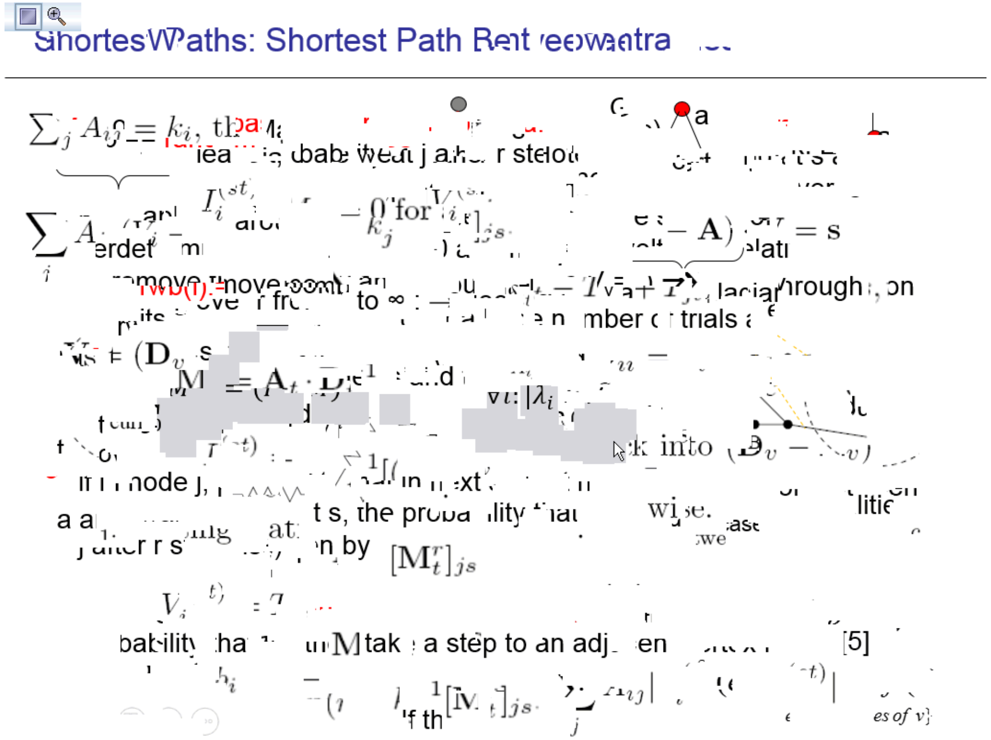


ShortesWPaths: Shortest Path Rent 'eowaatra

ShortesWPaths: Shortest Path Ret 'eewaatra

$\sum_j A_{ij} = k_i$, th

iea ... babi weat j a.h... r stelo

$\sum A$ ... erdet ... m ... I_i^{st} ... $k_j^{-1}[\mathbf{M}_t^r]_{js}$ ... $- \mathbf{A}) ... = \mathbf{s}$ ... lati

remove ... over r from 0 to ∞ : → geome ... T ... es → ... lacian through ... on

mits ... n mber ... Is a

$(\mathbf{D}_v$ ... of times $V_{j \to i}$ we go ... u ... averag ... d over

walks is

k into ... $\mathbf{D}_v$

in node j, ... $\mathbf{D}_t^{-1}[(\mathbf{I} - \mathbf{M}_t)^{-1}]_{js}$ ... lities

a a ... after r s ... ing ... at ... wise. ... case

n by $[\mathbf{M}_t^r]_{js}$

$\mathbf{V} = \mathbf{D}_t^{-1} \cdot (\mathbf{I} - \mathbf{M}_t)^{-1} \cdot \mathbf{s} = (\mathbf{D}_t$ ... $\mathbf{s}$.

bability that ... $\mathbf{M}$ take a step to an adjacent ... dge ... [5]

... $[\mathbf{M}_t]_{js}$ ... es of v}

net flow ... the ran ... ng the ... edge

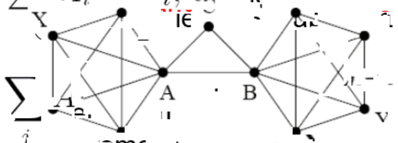
flow ... ough vertex

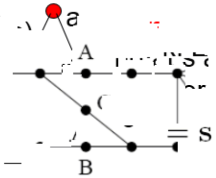
ShortesWPaths: Shortest Path Rent eewaatra

$\sum_j A_{ij} = k_i$, th

net flow through vertex i is a alf the s um f the flows

Shortie WPaths: hortest Path Rent eewaatra

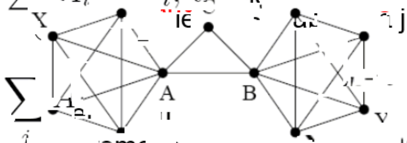


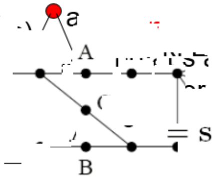


0.636	0.631	
0.200	0.982	
	0.068	
	0.269	
	0.004	
	0.024	

net flow through vertex i is a alf the s um f the flows

Shortie WPaths: hortest Path Rent eewaatra





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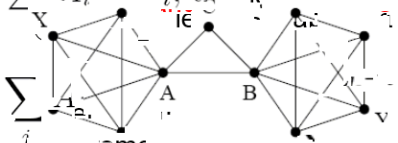

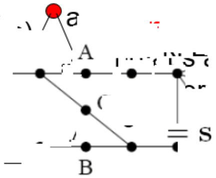

weighted adjacency matrix W

trality s(v) of node v is proportional to sum of centralities s(w) of nodes w (multiplied with corresp. edge weight)

0.636	0.631	
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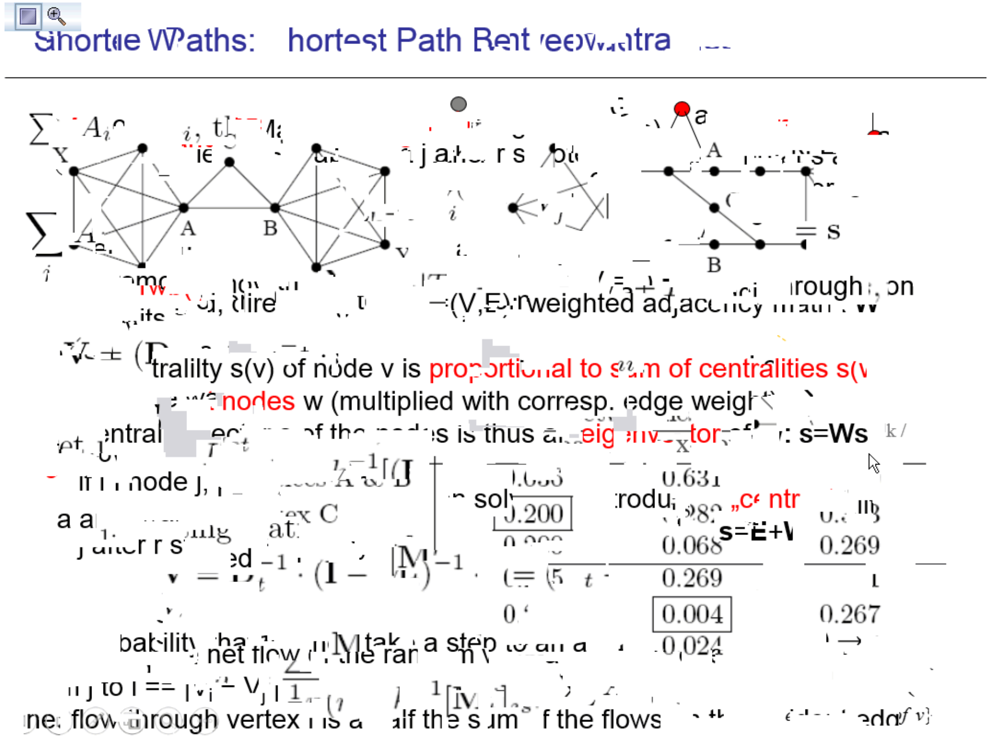


Shortde WPaths: hortest Path Bet eewaatra

(V,E) weighted adjacency matrix W

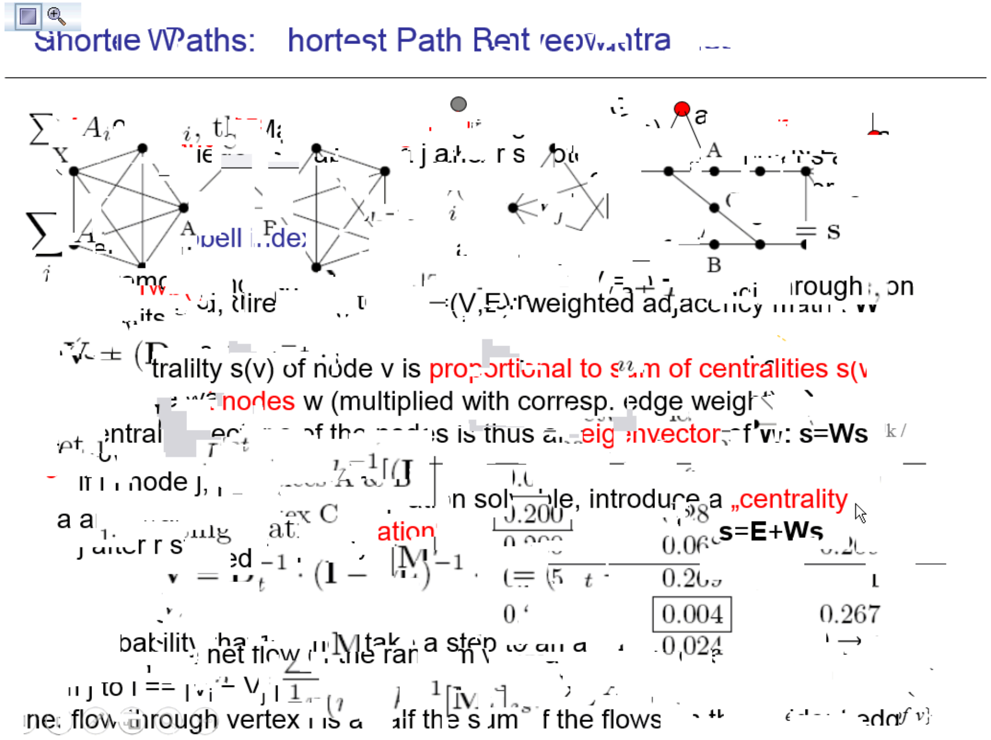
trality s(v) of node v is proportional to sum of centralities s(v

nodes w (multiplied with corresp. edge weight

entral ... of the nodes is thus an eigenvector of W: s=Ws

s=E+W

net flow through vertex i is a ... half the sum of the flows



Shortde WPaths: hortest Path Bet eewaatra

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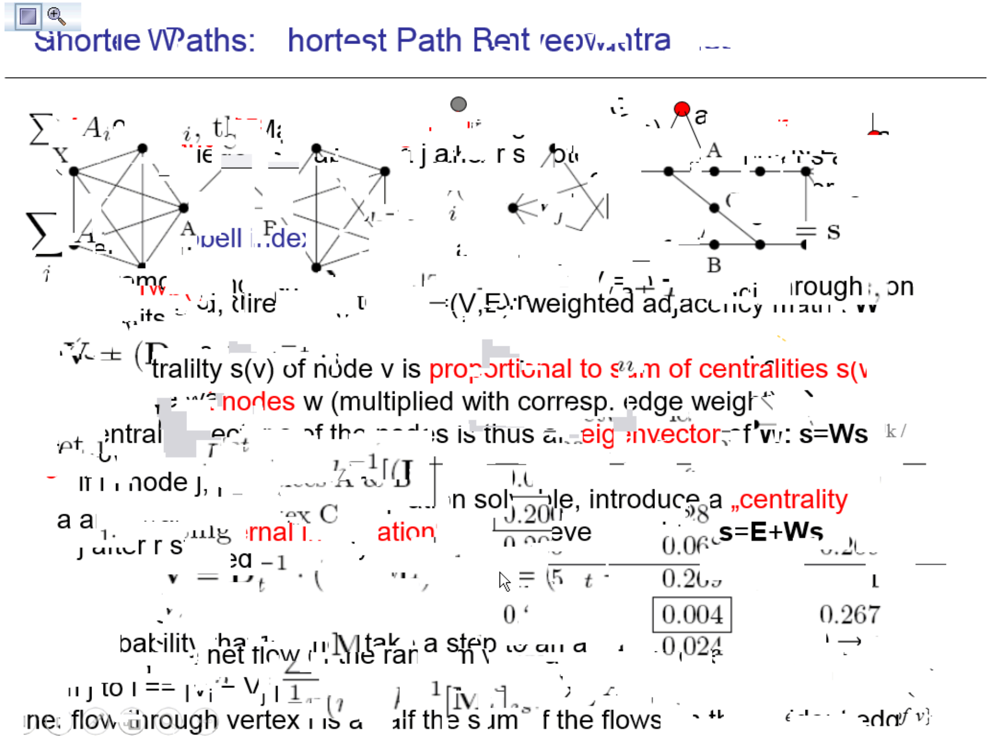

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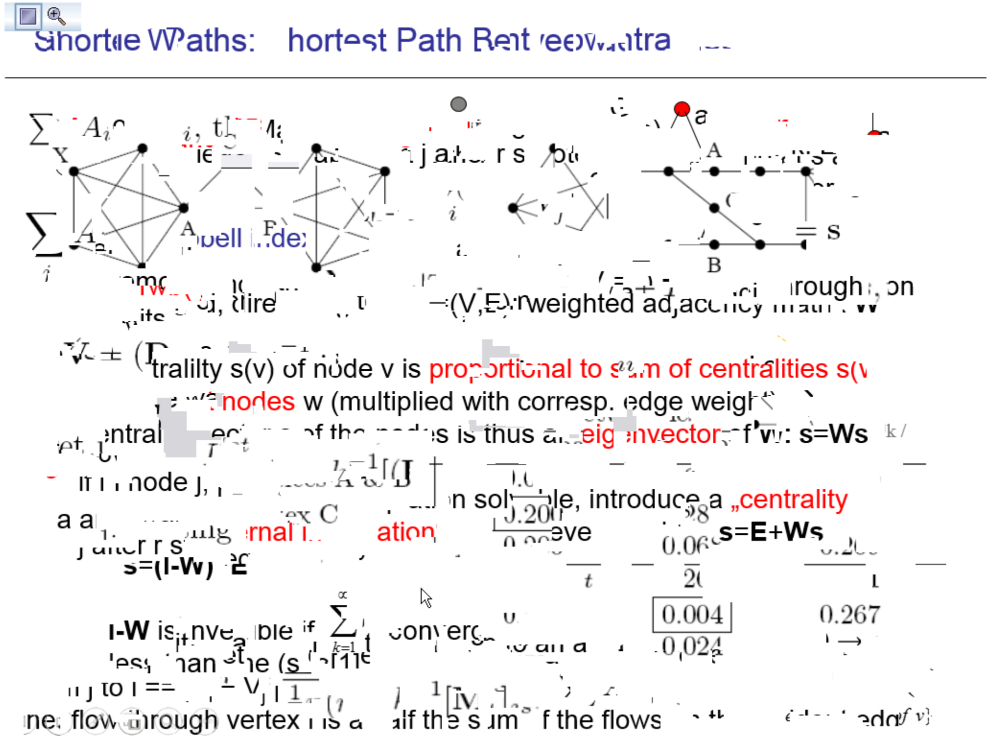

Shortde WPaths: hortest Path Bet eewaatra

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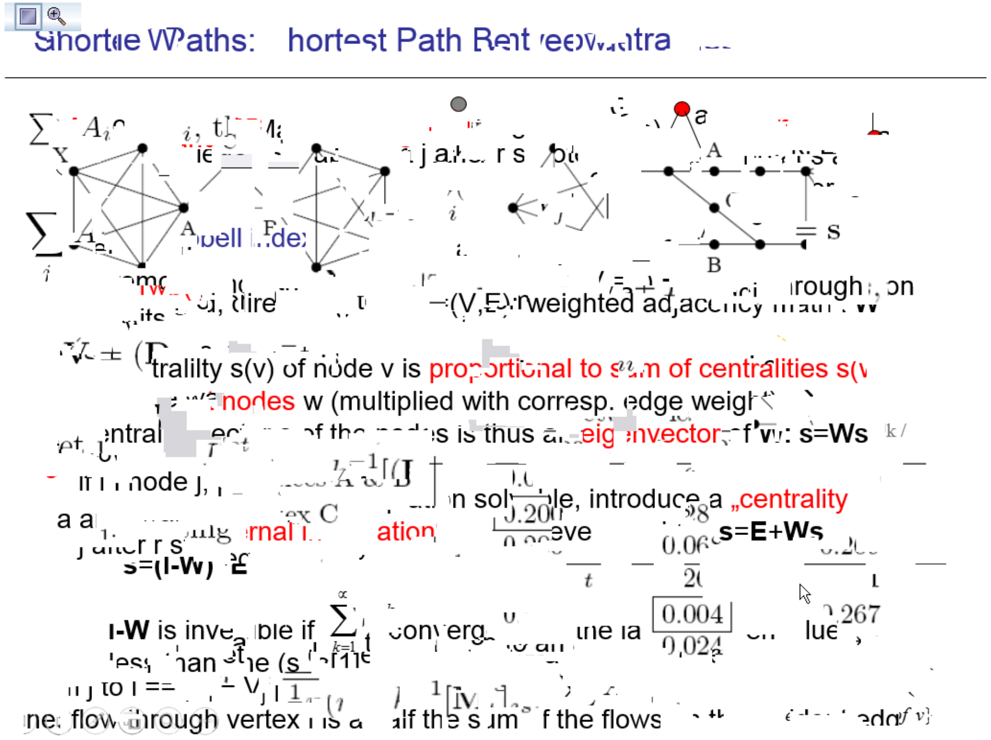
s=E+Ws

s=(I-W)$^{-1}$E

I-W is invertible if $\sum_{k=1}^{\infty}$ ... converges

net flow through vertex i is a ... half the sum of the flows



Shortest Paths: hortest Path Rent eew..tra

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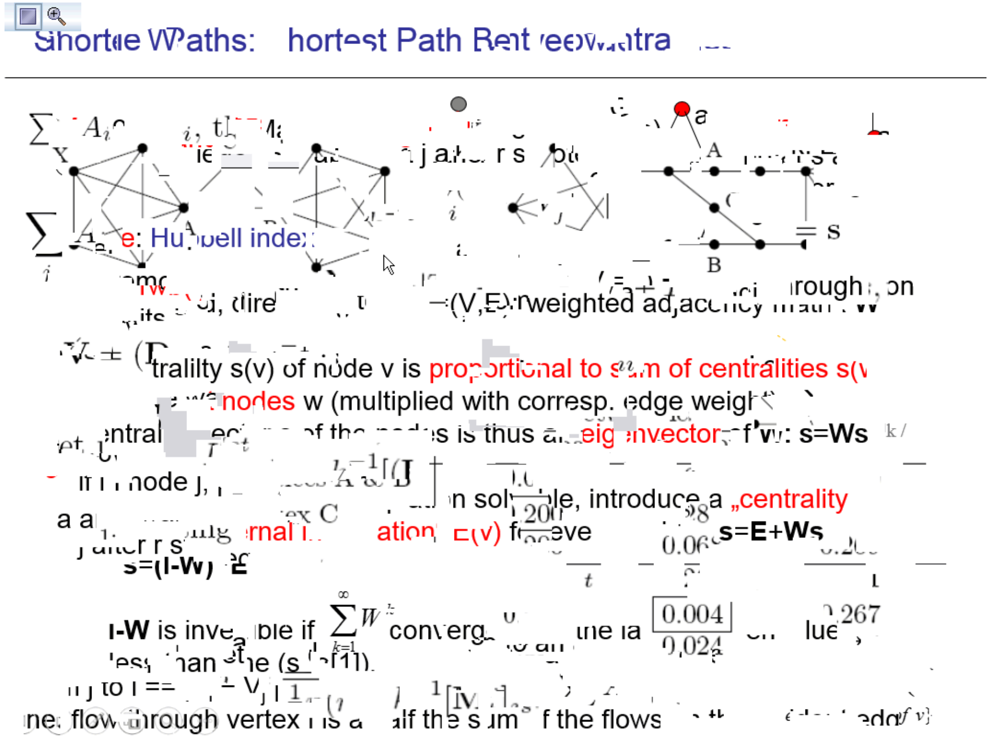
rnal ation eve

s=E+Ws

s=(I-W)⁻¹E

I-W is invertible if $\sum_{k=1}^{\infty} W^k$ converges

flow through vertex i is half the sum of the flows



e: Hubbell index

Shortest Paths: hortest Path Rent eew..tra

(V,E) weighted adjacency matrix W

trality s(v) of node v is proportional to sum of centralities s(v

nodes w (multiplied with corresp. edge weight

ntral ec of the nodes is thus an eigenvector of W: s=Ws

n solvable, introduce a „centrality

rnal ation E(v) for eve

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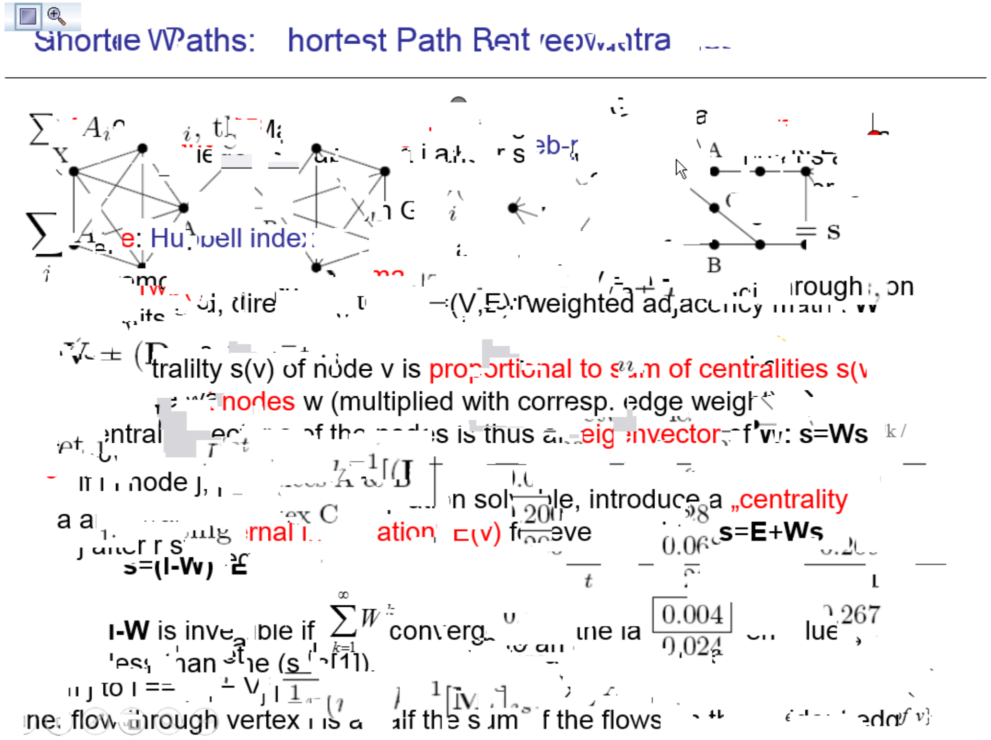

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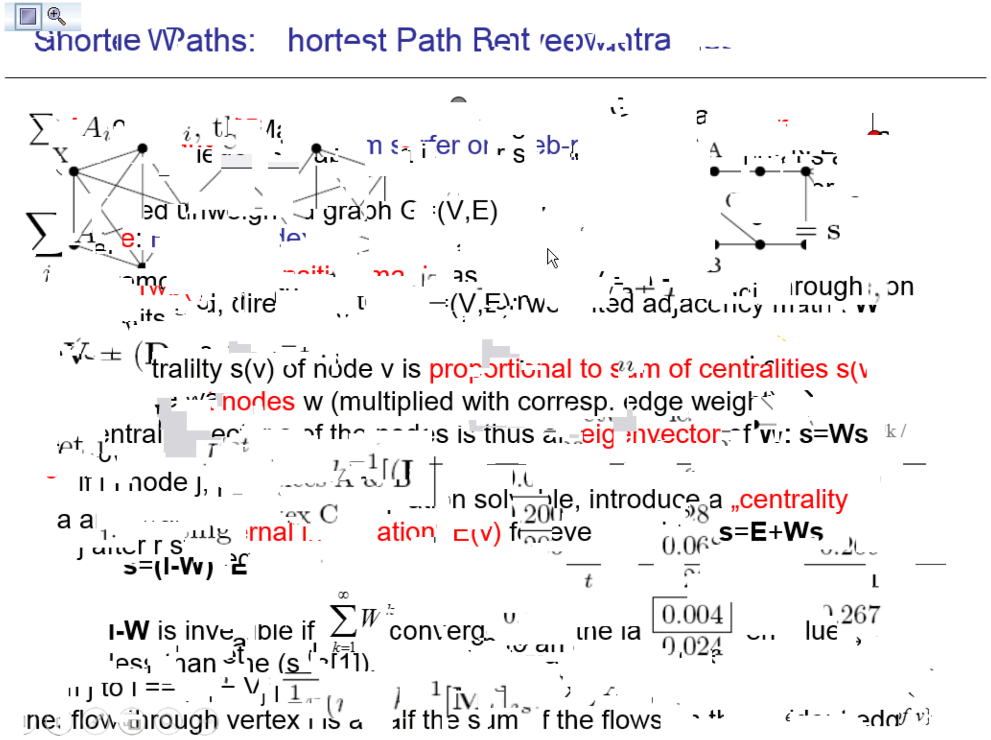

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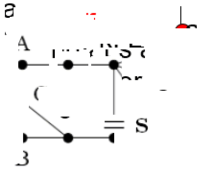

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$\boxed{0.004}$ 0,024 ... 0.267 ... lue

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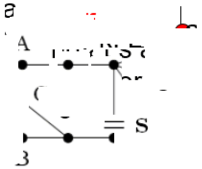

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in node j

$t_{ij} = \begin{cases} 0 & if\ (i,j) \\ \frac{1}{|V|} & if\ deg^+(i) \end{cases}$

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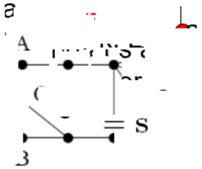

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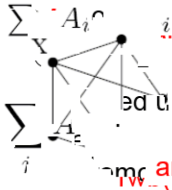

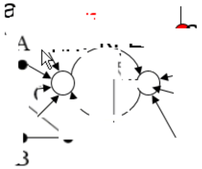

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going link randomly, pro ... rent node; if node is a

I-W is invertible if ... lue

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Shortest Paths: Shortest Path Rent ... Dijkstra

there a unique stationary distribution π? (
irreducible and positively recurrent?
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$$t_{ij} = \begin{cases} \frac{1}{\deg^+(i)} & \text{if } (i,j) \ldots \\ \frac{1}{|V|} & \text{if } \deg^+(i) \ldots \end{cases}$$
a „centrality
s=E+Ws
s=(I-W)$^{-1}$E
going link randomly, pro
I-W is inve... rent node; if node is a ... the la ... 0.004 ... 0.024 ... 0.267 ... lue
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net flow through vertex i is a ... alf the sum of the flows

Shortest Paths: Shortest Path Rent ... Dijkstra

there a unique stationary distribution π? (
irreducible and positively recurrent?
make it irreducible ... =αT+(1- α)E where E is the m...
to 1/n (completely stochastic choice).
... ted adjacency matrix W ... rough ... on
trality s(v ... ortional to ... m of centralities s(v
intronary distributions ↔ degr ... assume that: s=Ws
ghted graph with adjacency matrix A; we have then
$$t_{ij} = \frac{A_{ij}}{\deg(i)}$$
s=E+Ws
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$$t_{ij} = \frac{A_{ij}}{\deg(i)} \Rightarrow \pi_i = \frac{\deg(i)}{\sum \deg(v)}$$
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I-W is ... 0.004 ... 0.024 ... 0.267
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$$(\pi T)_j = \sum_{i\in V} \pi_i t_{ij} = \frac{\sum_{i\in V} \deg(i) t_{ij}}{\sum_{v\in V} \deg(v)} = \frac{\sum_{i\in V} A_{ij}}{\sum_{v\in V} \deg(v)} = \ldots = \pi_j$$
s=E+Ws
I-W is
net flow through vertex i is a ... alf the sum of the flows

Shortie WPaths: hortest Path Rent 'eow..atra

Is there a unique stationary distribution π? (

irreducible and positively recurrent?

graph

make it irreducible: T' =αT+(1- α)E where E is the m

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assume that s=Ws

ghted graph with adjacency matrix A; we have then

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s=ε+Ws

I-W is

$$(\pi T)_j = \sum_{i\in V}\pi_i t_{ij} = \frac{\sum_{i\in V}\deg(i)t_{ij}}{\sum_{v\in V}\deg(v)} = \frac{\sum_{i\in V}A_{ij}}{\sum_{v\in V}\deg(v)} = \frac{\deg(j)}{\sum_{v\in V}\deg(v)} = \pi_j$$

node; if node

net flow through vertex i is a ... alf the sum of the ...

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make it irreducible: T' =αT+(1- α)E where E is the m

ality of a web-page depends on the cen

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rough ... on

$$c(p) = d \sum_{q\in\{\text{In-neighbors of } p\}} \frac{c(q)}{\deg^+(q)} +$$

d is a ... factor, deg+(q) is the out

ntionary distributions ⟷ degr

assume that s=Ws

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Shortie WPaths: hortest Path Rent 'eow..atra

Is there a unique stat ... ary distribution π? (

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$= d\,\mathbf{P}\mathbf{c} + (... 1)$

make it irreducible: T' =αT+(1- α)E where E is the m

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rough ... on

$$c(p) = d \sum_{q\in\{\text{In-neighbors of } p\}} \frac{c(q)}{\deg^+(q)} + (1-d)$$

solution

d is a ... factor, deg+(q) is the out

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assume that s=Ws

ghted graph with adjacency matrix A; we have then

ation

$$\mathbf{c} = d\,\mathbf{P}\mathbf{c} + (1-d)(1, \ldots)$$

ality

s=ε+Ws

I-W is

$p_{ij} = 1/\deg(i)$

$$(\pi T)_j = \frac{\sum_{i\in V}\deg(i)t_{ij}}{\sum_{v\in V}\deg(v)} = \frac{\sum_{i\in V}A_{ij}}{\sum_{v\in V}\deg(v)} = \frac{\deg(j)}{\sum_{v\in V}\deg(v)} = \pi_j$$

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the solution a ... using matrix in ... Jacobi

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Shortte Wpaths: hortest Path Rent 'eewatra

...e a unique stationary distribution π? (

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$= d\,\mathbf{Pc} + (\quad 1)$

αT+(1-α)E ... E is t

ality of a web- ... depends on the cen... (compl... choice).

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$c(\,) = d \sum \frac{c(q)}{\deg^{+}(q)} + (1-d)$

the solution a ... matrix ... version ... Jacobi

d is a ...

$c_i^{(k+1)} = \sum_j P_{ij} c_j^{(k)} +$ ... have ... n

ality

$t_{ij} =$ $\mathbf{c} = d\,\mathbf{Pc} + (1-d)(1,$ ... s−E+Ws

I-W is

$(\pi T)_j$ ... $=1/\ldots \sum \deg(i) t_{ij} \ldots = \frac{\sum_{i\in V} A_{ij}}{\sum_{v\in V} \deg(v)} = \frac{\deg(j)}{\sum_{v\in} \deg(v)} = \pi_j$

net flow through vertex i is a half the sum of the ...

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I-W is

$(\pi T)_j$ ... $(\sum_{j<i} P_{ij} c_j^{(k+1)} + \sum_{j\geq i} \ldots c_j^{(k)}) + \ldots = \frac{\deg(j)}{\sum_{v\in} \deg(v)} = \pi_j$

net flow through vertex i is a half th... of the ...