Script **generated by TTT**

Title:	Seidl: Programmoptimierung (21.01.2016)
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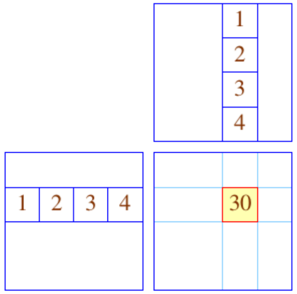
Example: Matrix-Matrix Multiplication

```
for $(i = 0; i < N; i{+}{+})$
    for $(j = 0; j < M; j{+}{+})$
        for $(k = 0; k < K; k{+}{+})$
            $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
```

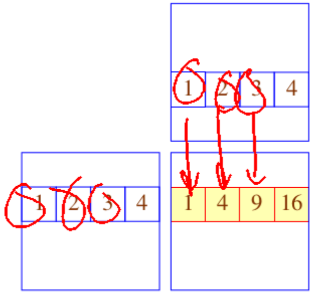
Over $b[][]$ the iteration is columnwise.



Exchange the two inner loops

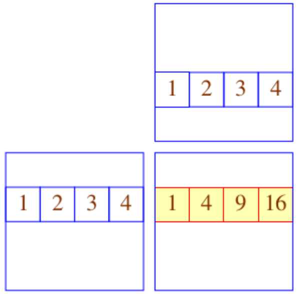
```
for $(i = 0; i < N; i{+}{+})$
    for $(k = 0; k < K; k{+}{+})$
        for $(j = 0; j < M; j{+}{+})$
            $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
```

Is this permitted ???



Discussion

- Correctness follows as before.
- A similar idea can also be used for the implementation of multiplication for row compressed matrices.
- Sometimes, the program must be massaged such that the transformation becomes applicable.
- Matrix-matrix multiplication perhaps requires initialization of the result matrix first ...



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for $(i = 0; i < N; i++)$
    for $(j = 0; j < M; j++)$ {
        $c[i][j] = 0;$
        for $(k = 0; k < K; k++)$
            $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
    }
```

- Now, the two iterations can no longer be exchanged.
- The iteration over j, however, can be duplicated ...

We obtain:

```
for $(i = 0; i < N; i++)$ {
    for $(j = 0; j < M; j++)$ $c[i][j] = 0;$
    for $(k = 0; k < K; k++)$
        for $(j = 0; j < M; j++)$
            $c[i][j] = c[i][j] + a[i][k] \cdot b[k][j];$
}
```

Discussion

- Instead of fusing several loops, we now have distributed the loops.
- Accordingly, conditionals may be moved out of the loop
  $\Longrightarrow$ if-distribution ...

We obtain:

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Discussion

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Caveat

Instead of using this transformation, the inner loop could also be optimized as follows:

```
for $(i = 0; i < N; i++)$
    for $(j = 0; j < M; j++)$ {
        $t = 0;$
        for $(k = 0; k < K; k++)$
            $t = t + a[i][k] \cdot b[k][j];$
        $c[i][j] = t;$
    }
```

Idea

If we find heavily used array elements $a[e_1] \ldots [e_r]$ whose index expressions stay constant within the inner loop, we could instead also provide auxiliary registers.

Caveat

The latter optimization prohibits the former and vice versa ...

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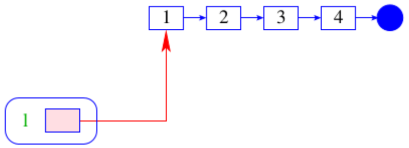
Discussion

- so far, the optimizations are concerned with iterations over arrays.
- Cache-aware organization of other data-structures is possible, but in general not fully automatic ...

Example: Stacks



Disadvantage

– The data-structure is bounded.

Improvement

- If the array is full, replace it with another of double size !!!
- If the array drops empty to a quarter, halve the array again !!!

⟹ The extra amortized costs are constant.

⟹ The implementation is no longer so trivial.

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Discussion

→ The same idea also works for queues.

→ Other data-structures are attempted to organize blockwise.

Problem: how can accesses be organized such that they refer mostly to the same block ???

⟹ Algorithms for external data

2. Stack Allocation instead of Heap Allocation

Problem

- Programming languages such as Java allocate all data-structures in the heap — even if they are only used within the current method.
- If no reference to these data survives the call, we want to allocate these on the stack.

⟹ Escape Analysis

Idea

Determine points-to information.

Determine if a created object is possibly reachable from the out side ...

Example: Our Pointer Language

$$x = \mathsf{new}();$$
$$y = \mathsf{new}();$$
$$x[A] = y;$$
$$z = y;$$
$$\mathsf{ret} = z;$$

... could be a possible method body.

Accessible from the outside world are memory blocks which:

- are assigned to a global variable such as ret; or
- are reachable from global variables.

... in the Example:

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We conclude:

- The objects which have been allocated by the first new() may never escape.
- They can be allocated on the stack.

Caveat

This is only meaningful if only few such objects are allocated during a method call.

If a local new() occurs within a loop, we still may allocate the objects in the heap.

Extension: Procedures

- We require an interprocedural points-to analysis.
- We know the whole program, we can, e.g., merge the control-flow graphs of all procedures into one and compute the points-to information for this.
- Caveat: If we always use the same global variables $y_1, y_2, \ldots$ for (the simulation of) parameter passing, the computed information is necessarily imprecise.
- If the whole program is not known, we must assume that each reference which is known to a procedure escapes.

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3.4 Wrap-Up

We have considered various optimizations for improving hardware utilization.

Arrangement of the Optimizations:

- First, global restructuring of procedures/functions and of loops for better memory behavior.
- Then local restructuring for better utilization of the instruction set and the processor parallelism.
- Then register allocation and finally,
- Peephole optimization for the final kick ...

Procedures:	Tail Recursion + Inlining Stack Allocation
Loops:	Iteration Reordering → if-Distribution → for-Distribution Value Caching
Bodies:	Life-Range Splitting (SSA) Instruction Scheduling with → Loop Unrolling → Loop Fusion
Instructions:	Register Allocation Instruction Selection Peephole Optimization

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4 Optimization of Functional Programs

Example:

$$\begin{aligned} \text{let rec } \mathsf{fac}\ x \quad = \quad & \text{if } x \leq 1 \text{ then } 1 \\ & \text{else } x \cdot \mathsf{fac}\ (x-1) \end{aligned}$$

- There are no basic blocks.
- There are no loops.
- Virtually all functions are recursive!

Strategies for Optimization:

⟹ Improve specific inefficiencies such as:

- Pattern matching
- Lazy evaluation (if supported)
- Indirections — Unboxing / Escape Analysis
- Intermediate data-structures — Deforestation

⟹ Detect and/or generate loops with basic blocks!

- Tail recursion
- Inlining
- **let**-Floating

Then apply general optimization techniques

... e.g., by translation into C.

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Warning:

Novel analysis techniques are needed to collect information about functional programs.

Example: Inlining

$$\begin{aligned} \text{let } \mathsf{max}\ (x, y) \quad &= \quad \text{if } x > y \text{ then } x \\ & \qquad \text{else } y \\ \text{let } \mathsf{abs}\ z \quad &= \quad \mathsf{max}\ (z, -z) \end{aligned}$$

As result of the optimization we expect ...

$$\begin{array}{lcll} \text{let } \mathsf{max}\,(x,y) & = & \textbf{if } x > y \textbf{ then } x & \\ & & \textbf{else } y & \\ \text{let } \mathsf{abs}\,z & = & \quad\text{let} & x = z \\ & & \text{in let} & y = -z \\ & & \text{in} & \boxed{\begin{array}{l} \textbf{if } x > y \textbf{ then } x \\ \textbf{else } y \end{array}} \end{array}$$

Discussion:

For the beginning, max is just a name. We must find out which value it takes at run-time

⟹ Value Analysis required !!

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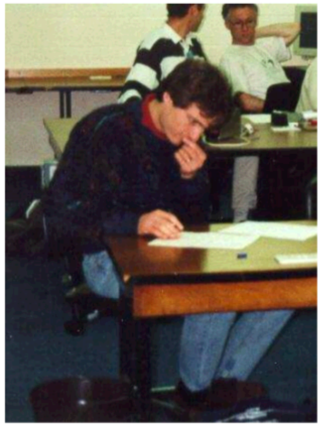

Nevin Heintze in the Australian team
of the Prolog-Programming-Contest, 1998

The complete picture:

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4.1 A Simple Functional Language

For simplicity, we consider:

$$\begin{array}{lcl} e & ::= & b \mid (e_1, \ldots, e_k) \mid c\, e_1 \ldots e_k \mid \mathbf{fun}\; x \to e \\ & & \mid (e_1\, e_2) \mid (\Box_1\, e) \mid (e_1\, \Box_2\, e_2) \mid \\ & & \mathbf{let}\; x_1 = e_1\; \mathbf{in}\; e_0 \mid \\ & & \mathbf{match}\; e_0\; \mathbf{with}\; p_1 \to e_1 \mid \ldots \mid p_k \to e_k \\ p & ::= & b \mid x \mid c\, x_1 \ldots x_k \mid (x_1, \ldots, x_k) \\ t & ::= & \mathbf{let\; rec}\; x_1 = e_1\; \mathbf{and} \ldots \mathbf{and}\; x_k = e_k\; \mathbf{in}\; e \end{array}$$

where b is a constant, x is a variable, c is a (data-)constructor and $\Box_i$ are i-ary operators.

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let max x y =

if x > y then x

else y

$: \underset{\uparrow}{'a} \to (\underset{\uparrow}{'a} \to \underset{\uparrow}{\text{bool}})$

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... in the Example:

A definition of max may look as follows:

$$\begin{array}{l} \mathbf{let}\; \mathsf{max} \;=\; \mathbf{fun}\; x \to \quad \mathbf{match}\; x \;\mathbf{with}\; (x_1, x_2) \;\to\; (\\ \qquad\qquad\qquad\qquad \mathbf{match}\; x_1 < x_2 \\ \qquad\qquad\qquad\qquad \;\;\mathbf{with}\;\; \mathsf{True} \;\to\; x_2 \\ \qquad\qquad\qquad\qquad \quad\mid \quad \mathsf{False} \;\to\; x_1 \\ \qquad\qquad\qquad\qquad) \end{array}$$

let max x y =
if x > y then x
else y

$: \; 'a \to ('a \to bool)$

Discussion

- **let rec** only occurs on top-level.
- Functions are always unary. Instead, there are explicit tuples.
- **if**-expressions and case distinction in function definitions is reduced to **match**-expressions.
- In case distinctions, we allow just simple patterns.

  ⟹ Complex patterns must be decomposed ...
- **let**-definitions correspond to basic blocks.
- Type-annotations at variables, patterns or expressions could provide further useful information

  — which we ignore.

if x > y then x else y

case x > y with true → x | false → y

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\qquad\qquad\qquad\qquad\quad \mid\;\; \text{False} \to x_1 \\
\qquad\qquad\qquad\quad)
\end{array}$$

Accordingly, we have for abs :

$$\begin{array}{lll} \textbf{let}\ \text{abs} & = \ \textbf{fun}\ x \rightarrow & \textbf{let}\ z = (x, -x) \\ & & \textbf{in}\ \text{max}\ z \end{array}$$

4.2 A Simple Value Analysis

Idea

For every subexpression e we collect the set $[e]^\sharp$ of possible values of e ...