Script **generated by TTT**

Title: Seidl: Programmoptimierung (23.12.2015)

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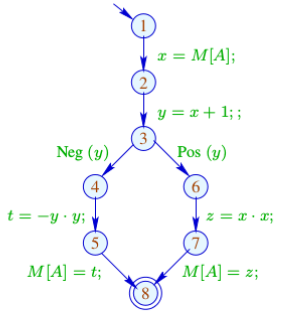
Duration: 89:13 min

Pages: 52

3.1 Registers

Example

$x = M[A];$
$y = x + 1;$
if (y) {
  $z = x \cdot x;$
  $M[A] = z;$
} else {
  $t = -y \cdot y;$
  $M[A] = t;$
}



The program uses 5 variables ...

Problem

What if the program uses more variables than there are registers.

Idea

Use one register for several variables.

In the example, e.g., one for x, t, z ...

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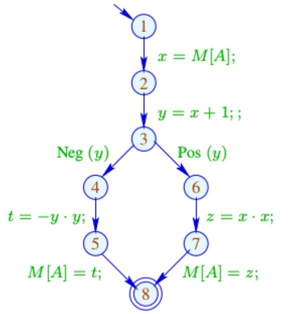

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The program uses 5 variables ...

Problem

What if the program uses more variables than there are registers.

Idea

Use one register for several variables.

In the example, e.g., one for x, t, z ...

$R = M[A];$
$y = R + 1;$
if (y) {
    $R = R \cdot R;$
    $M[A] = R;$
} else {
    $R = -y \cdot y;$
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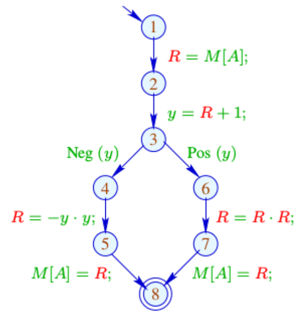
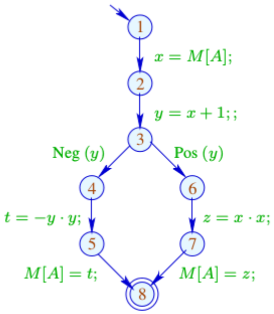
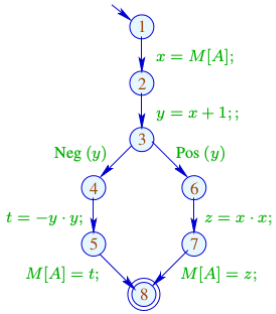
}





	$\mathcal{L}$
8	$\emptyset$
7	$\{A, z\}$
6	$\{A, x\}$
5	$\{A, t\}$
4	$\{A, y\}$
3	$\{A, x, y\}$
2	$\{A, x\}$
1	$\{A\}$
0	$\emptyset$



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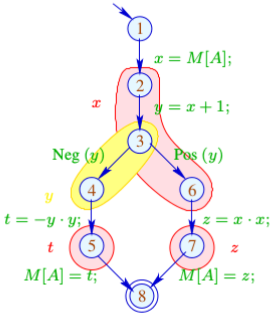


Live Ranges:

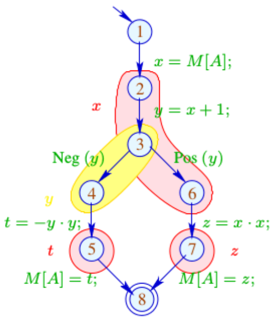
A	$\{0, \ldots, 7\}$
x	$\{2, 3, 6\}$
y	$\{2, 4\}$
t	$\{5\}$
z	$\{7\}$

In order to determine sets of compatible variables, we construct the Interference Graph $I = (\mathit{Vars}, E_I)$ where:

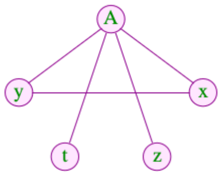
$$E_I = \{\{x, y\} \mid x \neq y, \mathcal{L}[x] \cap \mathcal{L}[y] \neq \emptyset\}$$

E_I has an edge for $x \neq y$ iff x, y are jointly live at some program point.

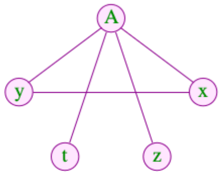
... in the Example:



Interference Graph:



Variables which are not connected with an edge can be assigned to the same register.



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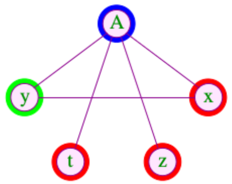
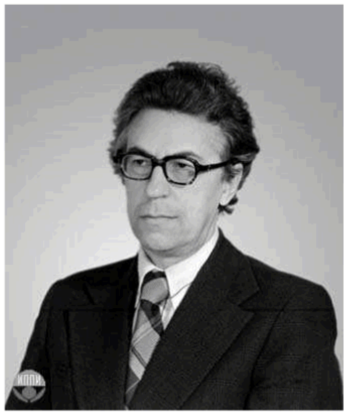


Color = Register

Sviatoslav Sergeevich Lavrov,
Russian Academy of Sciences (1962)



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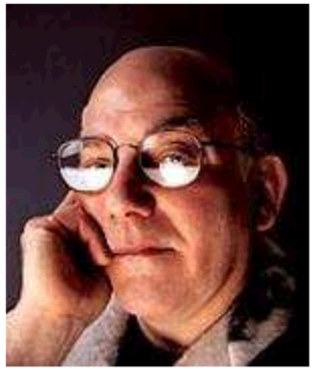


Gregory J. Chaitin, University of Maine (1981)

Abstract Problem

Given: Undirected Graph (V, E) .

Wanted: Minimal coloring, i.e., mapping $c : V \to \mathbb{N}$ mit

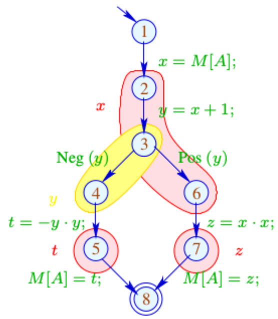
(1) $c(u) \neq c(v)$ for $\{u, v\} \in E$;

(2) $\bigsqcup\{c(u) \mid u \in V\}$ minimal!

- In the example, 3 colors suffice. But:
- In general, the minimal coloring is not unique.
- It is NP-complete to determine whether there is a coloring with at most k colors.

$\Longrightarrow$

We must rely on heuristics or special cases.



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Greedy Heuristics

- Start somewhere with color 1;
- Next choose the smallest color which is different from the colors of all already colored neighbors;
- If a node is colored, color all neighbors which not yet have colors;
- Deal with one component after the other ...

... more concretely:

```
forall (v ∈ V) c[v] = 0;
forall (v ∈ V) color (v);

void color (v) {
    if (c[v] ≠ 0) return;
    neighbors = {u ∈ V | {u,v} ∈ E};
    c[v] = ⨅{k > 0 | ∀ u ∈ neighbors : k ≠ c(u)};
    forall (u ∈ neighbors)
        if (c(u) == 0) color (u);
}
```

The new color can be easily determined once the neighbors are sorted according to their colors.

Discussion

→ Essentially, this is a Pre-order DFS.

→ In theory, the result may arbitrarily far from the optimum

→ ... in practice, it may not be as bad.

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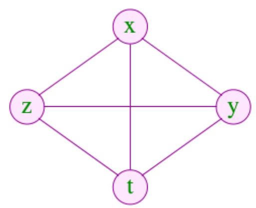
The algorithm works the better the smaller life ranges are ...

Idea: Life Range Splitting

Special Case: Basic Blocks

	$\mathcal{L}$
	x, y, z
$A_1 = x + y;$	x, z
$M[A_1] = z;$	x
$x = x + 1;$	x
$z = M[A_1];$	x, z
$t = M[x];$	x, z, t
$A_2 = x + t;$	x, z, t
$M[A_2] = z;$	x, t
$y = M[x];$	y, t
$M[y] = t;$	



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$t = M[x];$	x, z, t
$A_2 = x + t;$	x, z, t
$M[A_2] = z;$	x, t
$y = M[x];$	y, t
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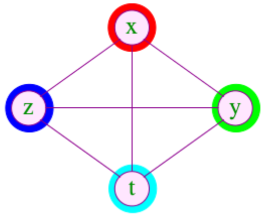

The live ranges of x and z can be split:

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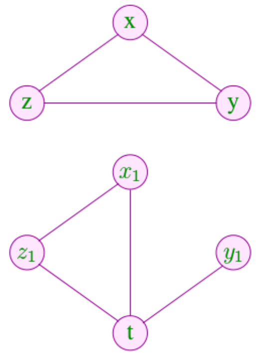
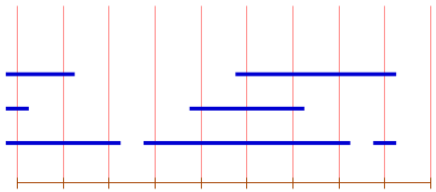
$M[A_1] = z;$	x
$x_1 = x + 1;$	x_1
$z_1 = M[A_1];$	x_1, z_1
$t = M[x_1];$	x_1, z_1, t
$A_2 = x_1 + t;$	x_1, z_1, t
$M[A_2] = z_1;$	x_1, t
$y_1 = M[x_1];$	y_1, t
$M[y_1] = t;$	



Interference graphs for minimal live ranges on basic blocks are known as interval graphs:



vertex $=$ interval

edge $=$ joint vertex

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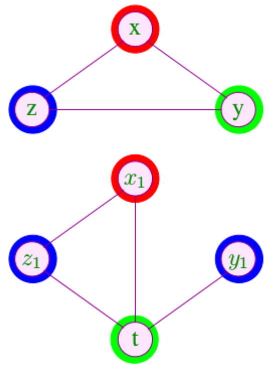

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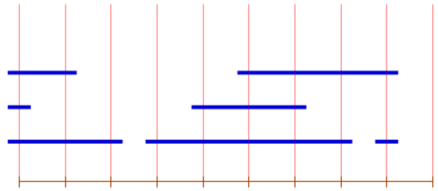

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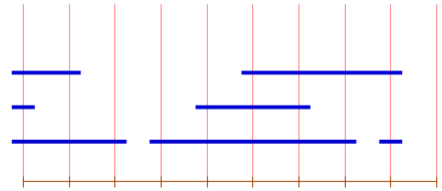

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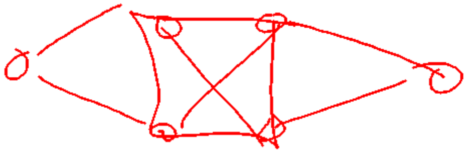
The covering number of a vertex is given by the number of incident intervals.



Theorem

maximal covering number

$=$ size of the maximal clique

$=$ minimally necessary number of colors

Graphs with this property (for every sub-graph) are called perfect ...

A minimal coloring can be found in polynomial time.

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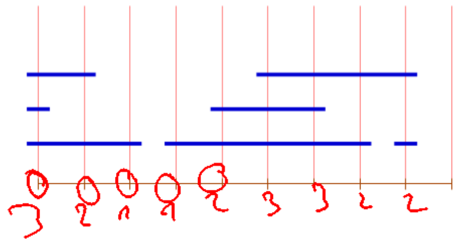

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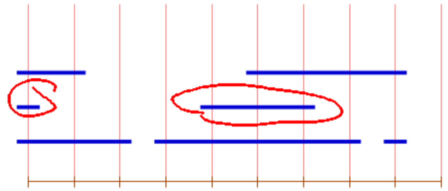

vertex	$=\!=$	interval
edge	$=\!=$	joint vertex

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A minimal coloring can be found in polynomial time.

Idea

- → Conceptually iterate over the vertices $0, \ldots, m-1$!
- → Maintain a list of currently free colors.
- → If an interval starts, allocate the next free color.
- → If an interval ends, free its color.

This results in the following algorithm:


```
free = [1, ..., k];
for (i = 0; i < m; i++) {
      init[i] = [];  exit[i] = [];
}
forall (I = [u, v] ∈ Intervals) {
      init[u] = (I :: init[u]);  exit[v] = (I :: exit[v]);
}
for (i = 0; i < m; i++) {
      forall (I ∈ init[i]) {
            color[I] = hd free;  free = tl free;
      }
      forall (I ∈ exit[i]) free = color[I] :: free;
}
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}
```


Discussion

→ For arbitrary programs, we thus may apply some heuristics for graph coloring ...

→ If the number of real register does not suffice, the remaining variables are spilled into a fixed area on the stack.

→ Generally, variables from inner loops are preferably held in registers.

→ For basic blocks we have succeeded to derive an optimal register allocation.

The number of required registers could even be determined before-hand !

→ This works only once live ranges have been split.

→ Splitting of live ranges for full programs results programs in static single assignment form ...

Discussion

- Every live variable should be defined at most once ??
- Every live variable should have at most one definition ?
- All definitions of the same variable should have a common end point !!!

⟹ Static Single Assignment Form

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⟹ Static Single Assignment Form

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How to arrive at SSA Form

We proceed in two phases:

Step 1:

Transform the program such that each program point v is reached by at most one definition of a variable x which is live at v.

Step 2:

- Introduce a separate variant x_i for every occurrence of a definition of a variable x !
- Replace every use of x with the use of the reaching variant x_h ...

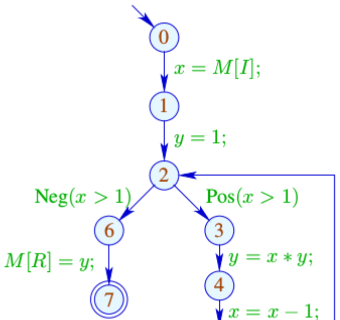
Implementing Step 1

- Determine for every program point the set of reaching definitions.
- **Assumption**

  All incoming edges of a join point v are labeled with the same parallel assignment $x = x \mid x \in L_v$ for some set L_v.

  Initially, $L_v = \emptyset$ for all v.
- If the join point v is reached by more than one definition for the same variable x which is live at program point v, insert x into L_v, i.e., add definitions $x = x;$ at the end of each incoming edge of v.

Example



Reaching Definitions

	$\mathcal{R}$
0	$\langle x, 0\rangle, \langle y, 0\rangle$
1	$\langle x, 1\rangle, \langle y, 0\rangle$
2	$\langle x, 1\rangle, \langle x, 5\rangle, \langle y, 2\rangle, \langle y, 4\rangle$
3	$\langle x, 1\rangle, \langle x, 5\rangle, \langle y, 2\rangle, \langle y, 4\rangle$
4	$\langle x, 1\rangle, \langle x, 5\rangle, \langle y, 4\rangle$
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Example

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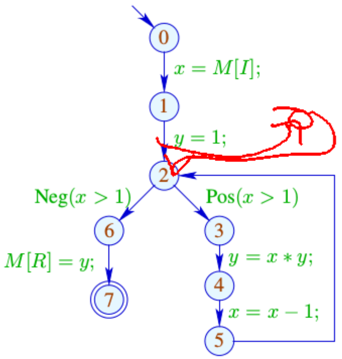

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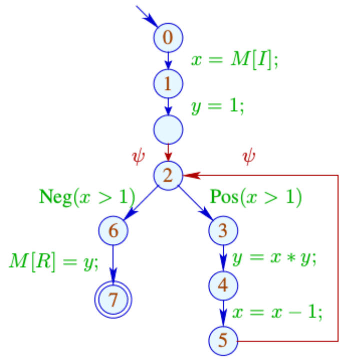

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$\langle x, 1\rangle, \langle y, 2\rangle$

where $\psi \quad \equiv \quad x = x \mid y = y$

Reaching Definitions

The complete lattice $\mathbb{R}$ for this analysis is given by:

$$\mathbb{R} = 2^{Defs}$$

where

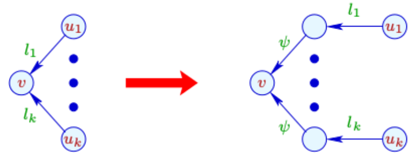
$$Defs = Vars \times Nodes \qquad Defs(x) = \{x\} \times Nodes$$

Then:

$$[\![(\_, x = r;, v)]\!]^\sharp R \quad = \quad R \backslash Defs(x) \cup \{\langle x, v\rangle\}$$
$$[\![(\_, x = x \mid x \in L, v)]\!]^\sharp R \quad = \quad R \backslash \bigcup_{x \in L} Defs(x) \cup \{\langle x, v\rangle \mid x \in L\}$$

The ordering on $\mathbb{R}$ is given by subset inclusion $\subseteq$ where the value at program start is given by $R_0 = \{\langle x, start\rangle \mid x \in Vars\}$.

The Transformation SSA, Step 1



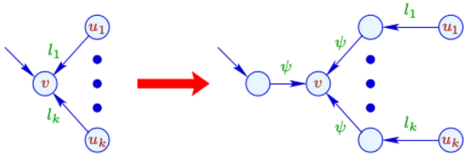
where $k \geq 2$.

The label ψ of the new in-going edges for v is given by:

$$\psi \quad \equiv \quad \{x = x \mid x \in \mathcal{L}[v], \#(\mathcal{R}[v] \cap Defs(x)) > 1\}$$

If the node v is the start point of the program, we add auxiliary edges whenever there are further ingoing edges into v:

The Transformation SSA, Step 1 (cont.)



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- Program start is interpreted as (the end point of) a definition of every variable x.
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Improvement

- We introduce assignments $x = x$ before v only if the sets of reaching definitions for x at incoming edges of v differ !
- This introduction is repeated until every v is reached by exactly one definition for each variable live at v.

Theorem

Assume that every program point in the controlflow graph is reachable from start and that every left-hand side of a definition is live. Then:

1. The algorithm for inserting definitions $x = x$ terminates after at most $n \cdot (m + 1)$ rounds were m is the number of program points with more than one in-going edges and n is the number of variables.
2. After termination, for every program point u, the set $\mathcal{R}[u]$ has exactly one definition for every variable x which is live at u.