Script generated by TTT

Title: Seidl: Programmoptimierung (10.12.2015)

Date: Thu Dec 10 08:34:28 CET 2015

Duration: 90:47 min

Pages: 45

Example: $f(x) = 3x^3 - 5x^2 + 4x + 13$

n	$f(n)$	Δ	Δ^2	Δ^3
0	13	2	8	18
1	15	10	26	
2	25	36		
3	61			
4				

Here, the n-th difference is always

$$\Delta_h^n(f) = n! \cdot a_n \cdot h^n \qquad (h \text{ step width})$$

Costs

- n times evaluation of f ;
- $\frac{1}{2} \cdot (n-1) \cdot n$ subtractions to determine the Δ^k ;
- n additions for every further value.

$\Longrightarrow$

Number of multiplications only depends on n.

Simple Case: $f(x) = a_1 \cdot x + a_0$

- ... naturally occurs in many numerical loops.
- The first differences are already constant:

$$f(x+h) - f(x) = a_1 \cdot h$$

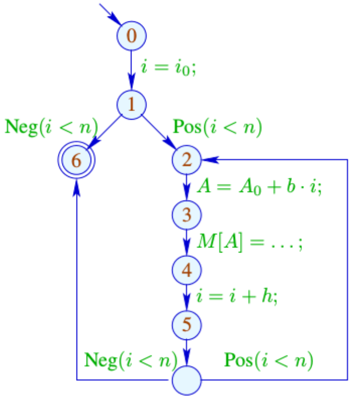
- Instead of the sequence: $y_i = f(x_0 + i \cdot h), \quad i \geq 0$

  we compute: $y_0 = f(x_0), \quad \Delta = a_1 \cdot h$

  $y_i = y_{i-1} + \Delta, \quad i > 0$

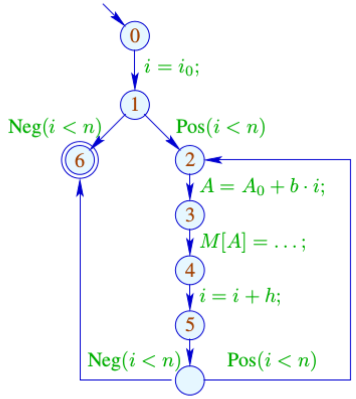
... or, after loop rotation:

$i = i_0;$
if $(i < n)$ do $\{$
$\quad A = A_0 + b \cdot i;$
$\quad M[A] = \ldots;$
$\quad i = i + h;$
$\}$ while $(i < n);$



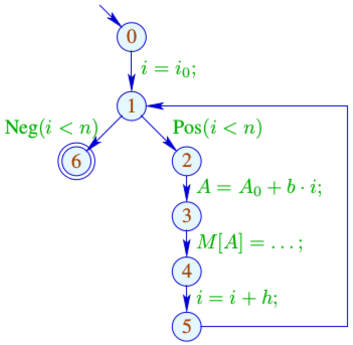
... or, after loop rotation:

$i = i_0;$
if $(i < n)$ do $\{$
$\quad A = A_0 + b \cdot i;$
$\quad M[A] = \ldots;$
$\quad i = i + h;$
$\}$ while $(i < n);$



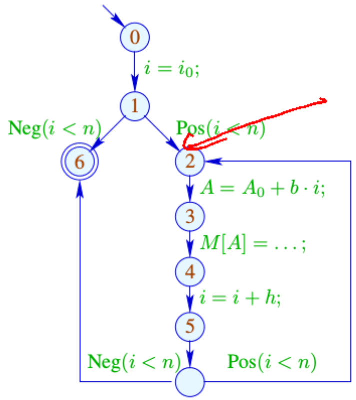
Example

for $(i = i_0; i < n; i = i + h)$ $\{$
$\quad A = A_0 + b \cdot i;$
$\quad M[A] = \ldots;$
$\}$



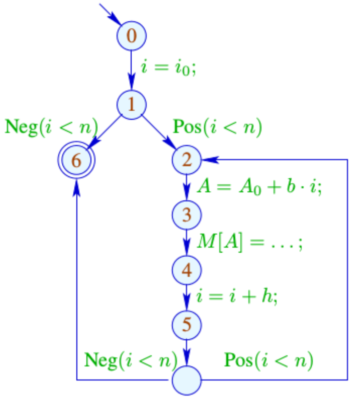
... or, after loop rotation:

$i = i_0;$
if $(i < n)$ do $\{$
$\quad A = A_0 + b \cdot i;$
$\quad M[A] = \ldots;$
$\quad i = i + h;$
$\}$ while $(i < n);$



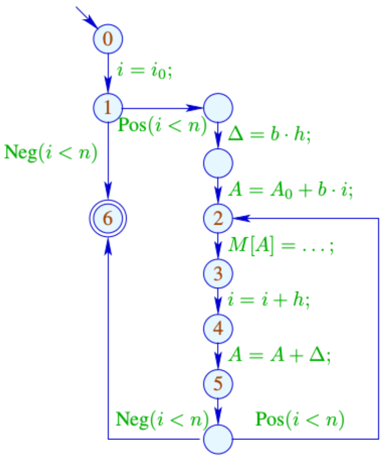
... or, after loop rotation:

$i = i_0;$
if $(i < n)$ do {
$\quad A = A_0 + b \cdot i;$
$\quad M[A] = \ldots;$
$\quad i = i + h;$
} while $(i < n);$



... and reduction of strength:

$i = i_0;$
if $(i < n)$ {
$\quad \Delta = b \cdot h;$
$\quad A = A_0 + b \cdot i_0;$
$\quad$ do {
$\qquad M[A] = \ldots;$
$\qquad i = i + h;$
$\qquad A = A + \Delta;$
$\quad$ } while $(i < n);$
}



Caveat

- The values b, h, A_0 must not change their values during the loop.
- i, A may be modified at exactly one position in the loop.
- One may try to eliminate the variable i altogether :

  → i may not be used else-where.

  → The initialization must be transformed into: $A = A_0 + b \cdot i_0$.

  → The loop condition $i < n$ must be transformed into: $A < N$ for $N = A_0 + b \cdot n$.

  → b must always be different from zero !!!

Caveat

- The values b, h, A_0 must not change their values during the loop.
- i, A may be modified at exactly one position in the loop.
- One may try to eliminate the variable i altogether :

  → i may not be used else-where.

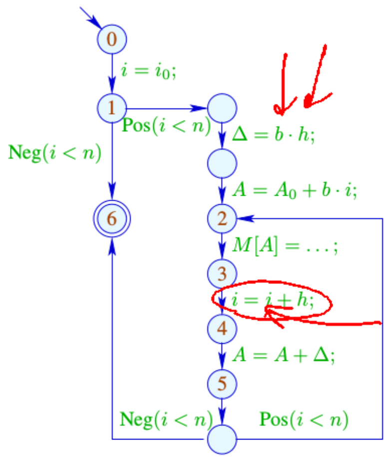
  → The initialization must be transformed into: $A = A_0 + b \cdot i_0$.

  → The loop condition $i < n$ must be transformed into: $A < N$ for $N = A_0 + b \cdot n$.

  → b must always be different from zero !!!

... and reduction of strength:

$i = i_0;$
if $(i < n)$ {
  $\Delta = b \cdot h;$
  $A = A_0 + b \cdot i_0;$
  do {
    $M[A] = \ldots;$
    $i = i + h;$
    $A = A + \Delta;$
  } while $(i < n);$
}

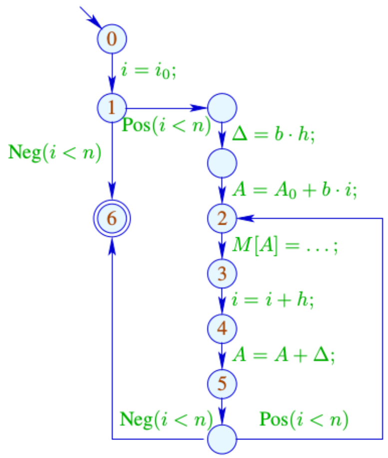


Caveat

- The values b, h, A_0 must not change their values during the loop.
- i, A may be modified at exactly one position in the loop.
- One may try to eliminate the variable i altogether :
  - → i may not be used else-where.
  - → The initialization must be transformed into: $A = A_0 + b \cdot i_0$.
  - → The loop condition $i < n$ must be transformed into: $A < N$ for $N = A_0 + b \cdot n$.
  - → b must always be different from zero !!!

... and reduction of strength:

$i = i_0;$
if $(i < n)$ {
  $\Delta = b \cdot h;$
  $A = A_0 + b \cdot i_0;$
  do {
    $M[A] = \ldots;$
    $i = i + h;$
    $A = A + \Delta;$
  } while $(i < n);$
}



Caveat

- The values b, h, A_0 must not change their values during the loop.
- i, A may be modified at exactly one position in the loop.
- One may try to eliminate the variable i altogether :
  - → i may not be used else-where.
  - → The initialization must be transformed into: $A = A_0 + b \cdot i_0$.
  - → The loop condition $i < n$ must be transformed into: $A < N$ for $N = A_0 + b \cdot n$.
  - → b must always be different from zero !!!

Caveat

- The values b, h, A_0 must not change their values during the loop.
- i, A may be modified at exactly one position in the loop.
- One may try to eliminate the variable i altogether :

  → i may not be used else-where.

  → The initialization must be transformed into: $A = A_0 + b \cdot i_0$.

  → The loop condition $i < n$ must be transformed into: $A < N$ for $N = A_0 + b \cdot n$.

  → b must always be different from zero !!!

Approach

Identify

... loops;

... iteration variables;

... constants;

... the matching use structures.

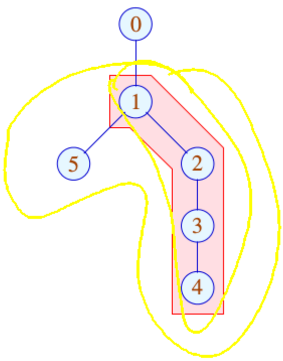
Loops:

... are identified through the node v with back edge $(\_, \_, v)$.

For the sub-graph G_v of the cfg on $\{w \mid v \Rightarrow w\}$, we define:

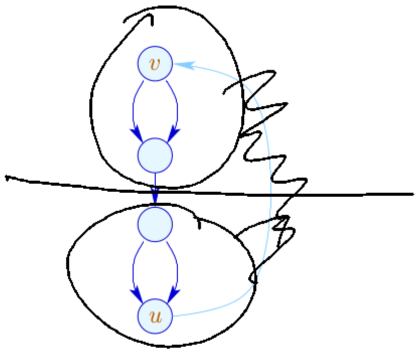
$$\text{Loop}[v] \quad = \quad \{w \mid w \rightarrow^* v \text{ in } G_v\}$$

Example



	$\mathcal{P}$
0	$\{0\}$
1	$\{0, 1\}$
2	$\{0, 1, 2\}$
3	$\{0, 1, 2, 3\}$
4	$\{0, 1, 2, 3, 4\}$
5	$\{0, 1, 5\}$

We are interested in edges which during each iteration are executed exactly once:



This property can be expressed by means of the pre-dominator relation ...

Assume that $(u, \_, v)$ is the back edge.

Then edges $k = (u_1, \_, v_1)$ could be selected such that:

- v pre-dominates u_1;
- u_1 pre-dominates v_1;
- v_1 predominates u

and is not contained in an inner loop.

Assume that $(u, \_, v)$ is the back edge.

Then edges $k = (u_1, \_, v_1)$ could be selected such that:

- v pre-dominates u_1;
- u_1 pre-dominates v_1;
- v_1 predominates u.

and is not contained in an inner loop.

On the level of source programs, this is trivial:

```
do { s_1 ... s_k
   } while (e);
```


The desired assignments must be among the s_i without preceeding jumps.

Iteration Variable:

i is an iteration variable if the only definition of i inside the loop occurs at an edge which separates the body and is of the form:

$$i = i + h;$$

for some loop constant h .

A loop constant is simply a constant (e.g., 42), or slightly more libaral, an expression which only depends on variables which are not modified during the loop.

Iteration Variable:

i is an iteration variable if the only definition of i inside the loop occurs at an edge which separates the body and is of the form:

$$i = i + h;$$

for some loop constant h .

A loop constant is simply a constant (e.g., 42), or slightly more libaral, an expression which only depends on variables which are not modified during the loop.

496

(3) Differences for Sets

Consider the fixpoint computation:

$$x = \emptyset;$$
$$\text{for } (t = F\,x; t \not\subseteq x; \boxed{t = F\,x;})$$
$$x = x \cup t;$$

If F is distributive, it could be replaced by:

$$x = \emptyset;$$
$$\text{for } (\Delta = F\,x; \Delta \neq \emptyset; \boxed{\Delta = (F\,\Delta) \setminus x;})$$
$$x = x \cup \Delta;$$

The function F must only be computed for the smaller sets Δ semi-naive iteration

Instead of the sequence: $\emptyset \subseteq F(\emptyset) \subseteq F^2(\emptyset) \subseteq \ldots$

we compute: $\Delta_1 \cup \Delta_2 \cup \ldots$

where:

$$\Delta_{i+1} = F(F^i(\emptyset)) \backslash F^i(\emptyset)$$
$$= F(\Delta_i) \backslash (\Delta_1 \cup \ldots \cup \Delta_i) \quad \text{with } \Delta_0 = \emptyset$$

Assume that the costs of $F\,x$ is $1 + \#x$

Then the costs may sum up to:

naive	$1 + 2 + \ldots + n + n$	$= \frac{1}{2}n(n+3)$
semi-naive		$2n$

where n is the cardinality of the result.

$\Longrightarrow$ A linear factor is saved.

2.2 Peephole Optimization

Idea

- Slide a small window over the program.
- Optimize agressively inside the window, i.e.,
  - → Eliminate redundancies!
  - → Replace expensive operations inside the window by cheaper ones!

Examples

$y = M[x]; x = x + 1; \quad \Longrightarrow \quad y = M[x{+}{+}];$

// given that there is a specific post-increment instruction

$z = y - a + a; \quad \Longrightarrow \quad z = y;$

// algebraic simplifications

$x = x; \quad \Longrightarrow \quad ;$

$x = 0; \quad \Longrightarrow \quad x = x \oplus x;$

$x = 2 \cdot x; \quad \Longrightarrow \quad x = x + x;$

Examples

$y = M[x]; x = x + 1; \quad \Longrightarrow \quad y = M[x{+}{+}];$

// given that there is a specific post-increment instruction

$z = y - a + a; \quad \Longrightarrow \quad z = y;$

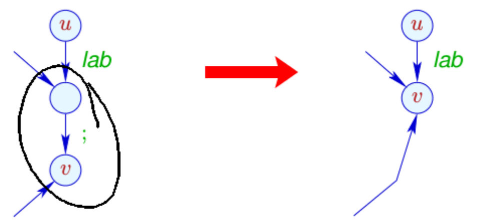
// algebraic simplifications

$x = x; \quad \Longrightarrow \quad ;$

$x = 0; \quad \Longrightarrow \quad x = x \oplus x;$

$x = 2 \cdot x; \quad \Longrightarrow \quad x = x + x;$

Important Subproblem: *nop*-Optimization



- → If $(v_1, ;, v)$ is an edge, v_1 has no further out-going edge.
- → Consequently, we can identify v_1 and v .
- → The ordering of the identifications does not matter.

Implementation

- We construct a function next : $Nodes \to Nodes$ with:

$$\text{next } u = \begin{cases} \text{next } v & \text{if } (u, ;, v) \text{ edge} \\ u & \text{otherwise} \end{cases}$$

Caveat: This definition is only recursive if there are ;-loops ???

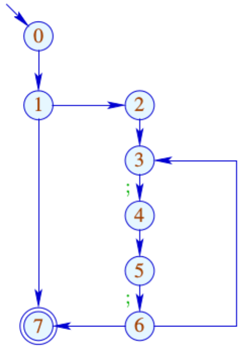
- We replace every edge:

$$(u, lab, v) \quad \Longrightarrow \quad (u, lab, \text{next } v)$$

... whenever $lab \neq ;$

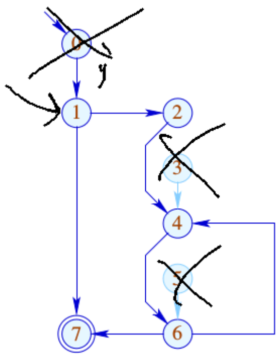
- All ;-edges are removed.

Example



next 1 = 1
next 3 = 4
next 5 = 6

Example



next 1 = 1
next 3 = 4
next 5 = 6

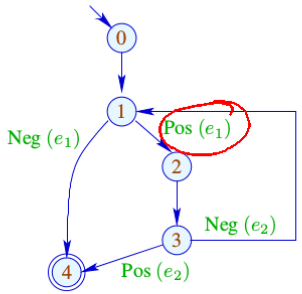
2. Subproblem: Linearization

After optimization, the CFG must again be brought into a linear arrangement of instructions.

Caveat

Not every linearization is equally efficient !!!

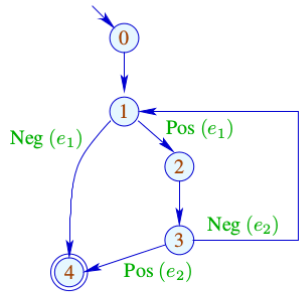
Example



0:
1: if (e_1) goto 2;
4: halt
2: Rumpf
3: if (e_2) goto 4;
goto 1;

Bad: The loop body is jumped into.

Example



0:
1: if $(!e_1)$ goto 4;
2: Rumpf
3: if $(!e_2)$ goto 1;
4: halt

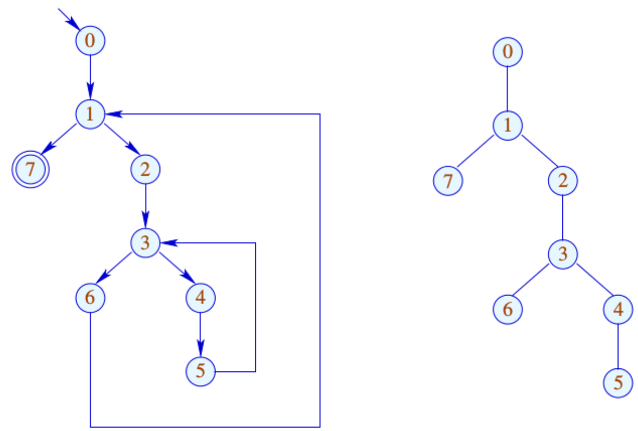
// better cache behavior

Idea

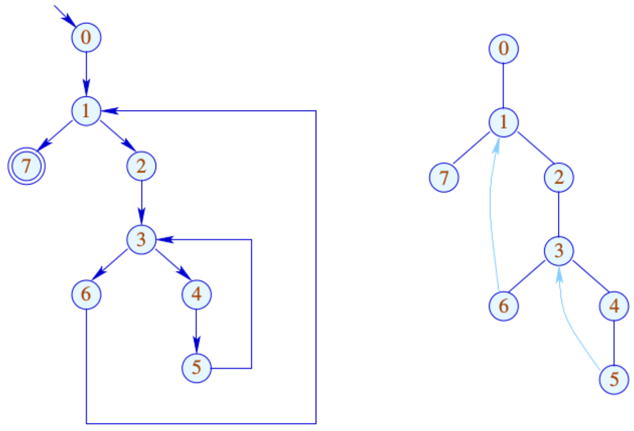
- Assign to each node a temperature!
- always jumps to
  - (1) nodes which have already been handled;
  - (2) colder nodes.
- Temperature $\approx$ nesting-depth

For the computation, we use the pre-dominator tree and strongly connected components ...

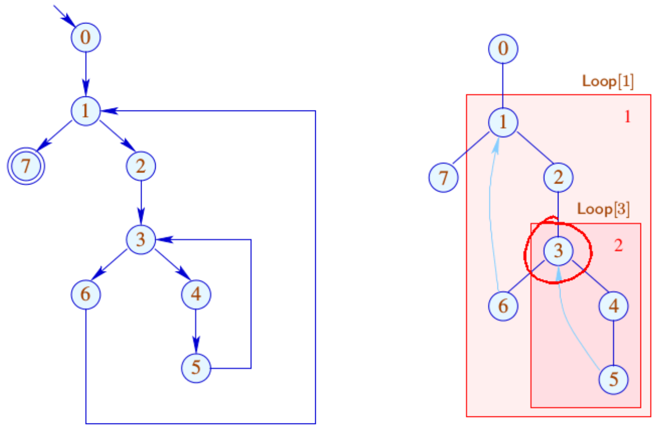
More Complicated Example



More Complicated Example

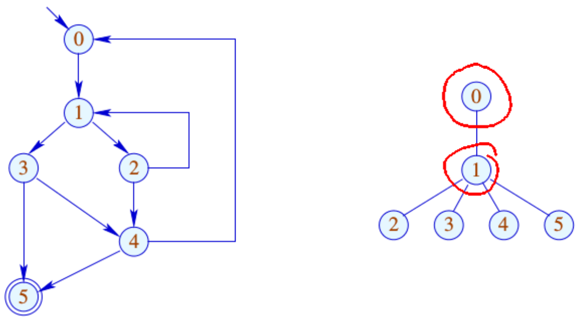


More Complicated Example



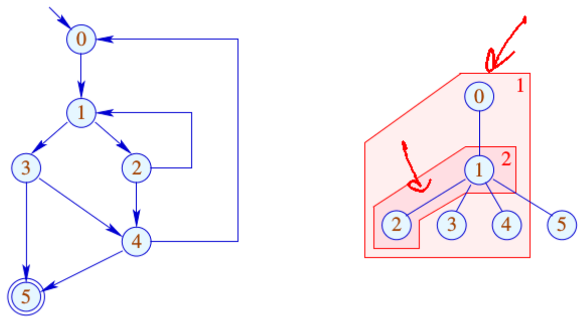
Our definition of Loop implies that (detected) loops are necessarily nested.

Is is also meaningful for do-while-loops with breaks ...



Our definition of Loop implies that (detected) loops are necessarily nested.

Is is also meaningful for do-while-loops with breaks ...



Summary: The Approach

(1) For every node, determine a temperature;

(2) Pre-order-DFS over the CFG;

- → If an edge leads to a node we already have generated code for, then we insert a jump.
- → If a node has two successors with different temperature, then we insert a jump to the colder of the two.
- → If both successors are equally warm, then it does not matter.

2.3 Procedures

We extend our mini-programming language by procedures without parameters and procedure calls.

For that, we introduce a new statement:

$$f();$$

Every procedure f has a definition:

$$f\,()\ \{\ stmt^*\ \}$$

Additionally, we distinguish between global and local variables.

Program execution starts with the call of a procedure main () .