Script generated by TTT

Title:	Seidl: Programmoptimierung (06.11.2013)
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Pages:	43

Summary and Application:

→ The effects of edges of the analysis of availability of expressions are distributive:

$$\begin{aligned}(a \cup (x_1 \cap x_2))\backslash b &= ((a \cup x_1) \cap (a \cup x_2))\backslash b \\ &= ((a \cup x_1)\backslash b) \cap ((a \cup x_2)\backslash b)\end{aligned}$$

→ If all effects of edges are distributive, then the MOP can be computed by means of the constraint system and RR-iteration. :-)

→ If not all effects of edges are distributive, then RR-iteration for the constraint system at least returns a safe upper bound to the MOP :-)

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1.2 Removing Assignments to Dead Variables

Example:

$$\begin{aligned} 1: &\quad x = y + 2; \\ 2: &\quad y = 5; \\ 3: &\quad x = y + 3; \end{aligned}$$

The value of x at program points $1, 2$ is over-written before it can be used.

Therefore, we call the variable x dead at these program points :-)

Note:

→ Assignments to dead variables can be removed ;-)

→ Such inefficiencies may originate from other transformations.

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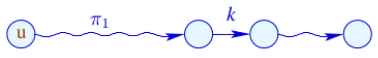
Formal Definition:

The variable x is called live at u along the path π starting at u relative to a set X of variables either:

if $x \in X$ and π does not contain a definition of x; or:

if π can be decomposed into: $\pi = \pi_1 k \pi_2$ such that:

- k is a use of x ; and
- π_1 does not contain a definition of x.

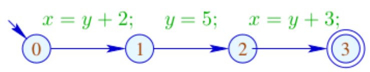


Thereby, the set of all defined or used variables at an edge $k = (\_, lab, \_)$ is defined by:

lab	used	defined
;	$\emptyset$	$\emptyset$
Pos (e)	$Vars\,(e)$	$\emptyset$
Neg (e)	$Vars\,(e)$	$\emptyset$
$x = e;$	$Vars\,(e)$	$\{x\}$
$x = M[e];$	$Vars\,(e)$	$\{x\}$
$M[e_1] = e_2;$	$Vars\,(e_1) \cup Vars\,(e_2)$	$\emptyset$

A variable x which is not live at u along π (relative to X) is called dead at u along π (relative to X).

Example:



where $X = \emptyset$. Then we observe:

	live	dead
0	$\{y\}$	$\{x\}$
1	$\emptyset$	$\{x, y\}$
2	$\{y\}$	$\{x\}$
3	$\emptyset$	$\{x, y\}$

The variable x is live at u (relative to X) if x is live at u along some path to the exit (relative to X). Otherwise, x is called dead at u (relative to X).

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Question:

How can the sets of all dead/live variables be computed for every u ???

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Idea:

For every edge $k = (u, \_, v)$, define a function $[\![k]\!]^\sharp$ which transforms the set of variables which are live at v into the set of variables which are live at u ...

Let $\mathbb{L} = 2^{Vars}$.

For $k = (\_, lab, \_)$, define $[\![k]\!]^\sharp = [\![lab]\!]^\sharp$ by:

$$
\begin{array}{lll}
[\![;]\!]^\sharp\, L & = & L \\
[\![\mathrm{Pos}(e)]\!]^\sharp\, L & = & [\![\mathrm{Neg}(e)]\!]^\sharp\, L \;=\; L \cup Vars(e) \\
[\![x = e;]\!]^\sharp\, L & = & (L \backslash \{x\}) \cup Vars(e) \\
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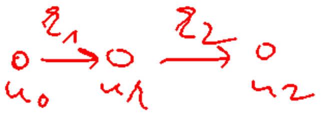
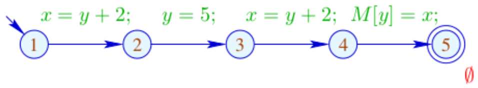
$[\![k]\!]^\sharp$ can again be composed to the effects of $[\![\pi]\!]^\sharp$ of paths $\pi = k_1 \ldots k_r$ by:

$$[\![\pi]\!]^\sharp = [\![k_1]\!]^\sharp \circ \ldots \circ [\![k_r]\!]^\sharp$$



We verify that these definitions are meaningful :-)



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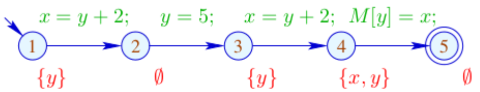


The set of variables which are live at u then is given by:

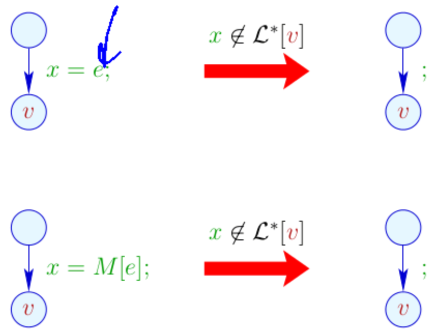
$$\mathcal{L}^*[u] \;=\; \bigcup \{ [\![\pi]\!]^\sharp\, X \mid \pi : u \to^* stop \}$$

... literally:

- The paths start in u :-)

  $\Longrightarrow$ As partial ordering for $\mathbb{L}$ we use $\sqsubseteq \,=\, \subseteq$.
- The set of variables which are live at program exit is given by the set X :-)

Transformation 2:



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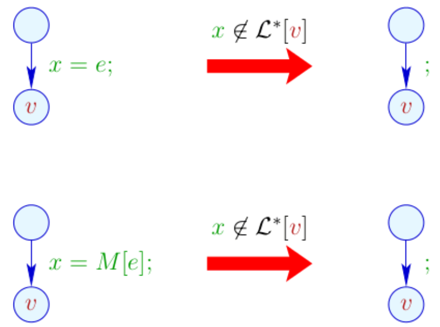

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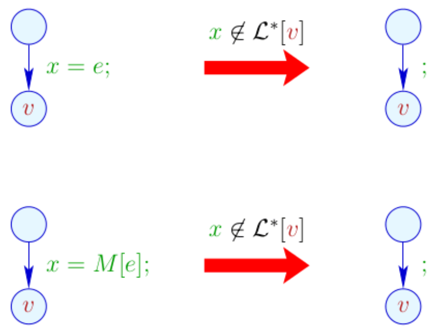

Correctness Proof:

→ Correctness of the effects of edges: If L is the set of variables which are live at the exit of the path π, then $[\![\pi]\!]^\sharp\, L$ is the set of variables which are live at the beginning of π :-)

→ Correctness of the transformation along a path: If the value of a variable is accessed, this variable is necessarily live. The value of dead variables thus is irrelevant :-)

→ Correctness of the transformation: In any execution of the transformed programs, the live variables always receive the same values :-))

Computation of the sets $\mathcal{L}^*[u]$:

(1) Collecting constraints:

$$\mathcal{L}[stop] \supseteq X$$
$$\mathcal{L}[u] \supseteq [\![k]\!]^\sharp (\mathcal{L}[v]) \qquad k = (u, \_, v) \quad \text{edge}$$

(2) Solving the constraint system by means of RR iteration.
Since $\mathbb{L}$ is finite, the iteration will terminate :-)

(3) If the exit is (formally) reachable from every program point, then the smallest solution $\mathcal{L}$ of the constraint system equals $\mathcal{L}^*$ since all $[\![k]\!]^\sharp$ are distributive :-))

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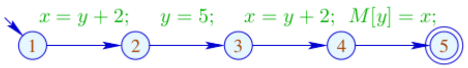

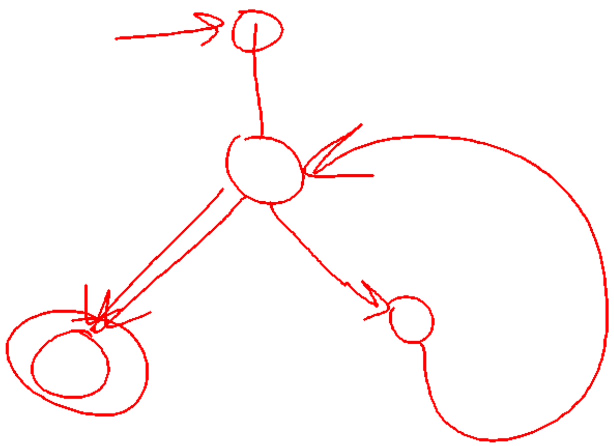

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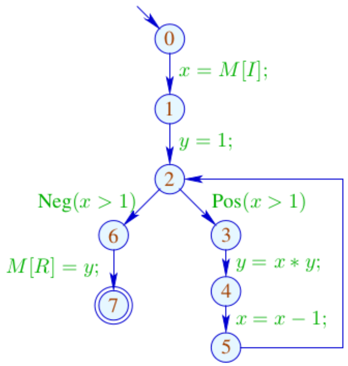
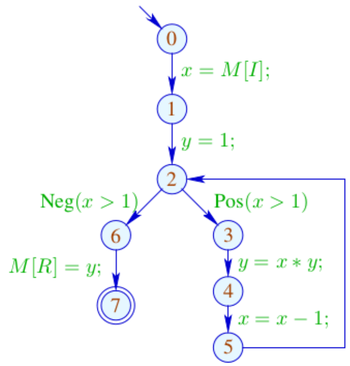
Caveat: The information is propagated backwards !!!

Example:



$$\mathcal{L}[0] \supseteq (\mathcal{L}[1]\backslash\{x\}) \cup \{I\}$$
$$\mathcal{L}[1] \supseteq \mathcal{L}[2]\backslash\{y\}$$
$$\mathcal{L}[2] \supseteq (\mathcal{L}[6] \cup \{x\}) \cup (\mathcal{L}[3] \cup \{x\})$$
$$\mathcal{L}[3] \supseteq (\mathcal{L}[4]\backslash\{y\}) \cup \{x, y\}$$
$$\mathcal{L}[4] \supseteq (\mathcal{L}[5]\backslash\{x\}) \cup \{x\}$$
$$\mathcal{L}[5] \supseteq \mathcal{L}[2]$$
$$\mathcal{L}[6] \supseteq \mathcal{L}[7] \cup \{y, R\}$$
$$\mathcal{L}[7] \supseteq \emptyset$$

Example:

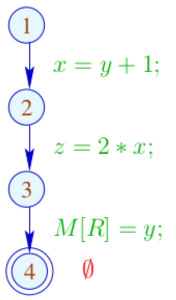


	1	2
7	$\emptyset$	
6	$\{y, R\}$	
2	$\{x, y, R\}$	dito
5	$\{x, y, R\}$	
4	$\{x, y, R\}$	
3	$\{x, y, R\}$	
1	$\{x, R\}$	
0	$\{I, R\}$	

The left-hand side of no assignment is dead :-)

Caveat:

Removal of assignments to dead variables may kill further variables:



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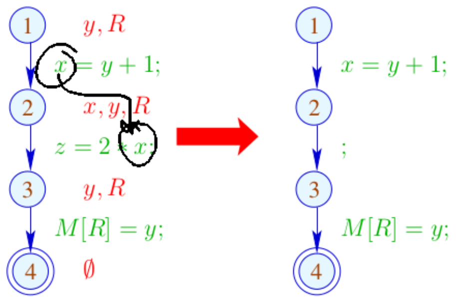

Re-analyzing the program is inconvenient :-(

Idea: Analyze true liveness!

x is called truly live at u along a path π (relative to X), either

if $x \in X$, π does not contain a definition of x; or

if π can be decomposed into $\pi = \pi_1\, k\, \pi_2$ such that:

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The set of truely used variables at an edge $k = (\_, lab, v)$ is defined as:

lab	truely used
;	$\emptyset$
Pos (e)	$Vars\,(e)$
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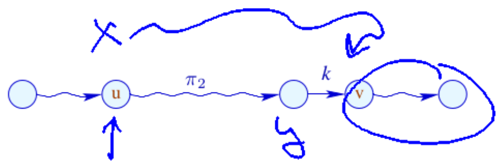
$(*)$ – given that x is truely live at v w.r.t. π_2 :-)



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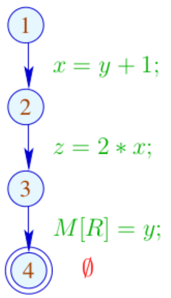

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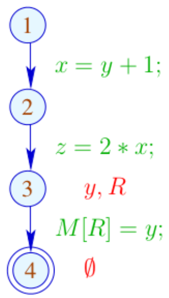

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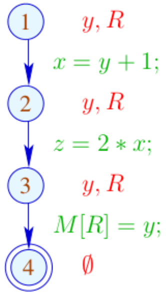

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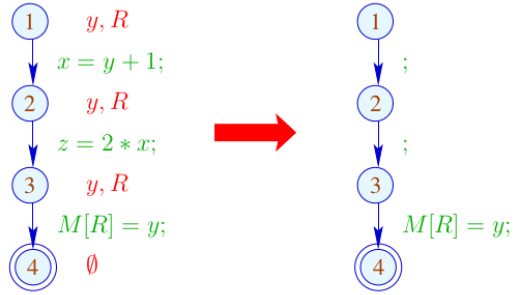

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To see this, consider for $\mathbb{D} = 2^U$, $f\,y = (u \in y)\,?\,b :\, \emptyset$ We verify:

$$\begin{aligned} f\,(y_1 \cup y_2) &= (u \in y_1 \cup y_2)\,?\,b :\, \emptyset \\ &= (u \in y_1 \vee u \in y_2)\,?\,b :\, \emptyset \\ &= (u \in y_1)\,?\,b :\, \emptyset \cup (u \in y_2)\,?\,b :\, \emptyset \\ &= f\,y_1 \cup f\,y_2 \end{aligned}$$

$a \subseteq b$

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$\Longrightarrow$ the constraint system yields the MOP :-))

- True liveness detects more superfluous assignments than repeated liveness !!!