

Script generated by TTT

Title: Seidl: Programoptimierung (06.02.2013)

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Remarks:

- Not all positive functions are monotonic !!!
- For k variables, there are $2^{2^k-1} + 1$ many functions.
- The height of the complete lattice is 2^k .
- We construct an interprocedural analysis which for every predicate p determines a (monotonic) transformation

$$[[p]]^\sharp : \text{Pos} \rightarrow \text{Pos}$$

- For every clause, $p(X_1, \dots, X_k) \Leftarrow g_1, \dots, g_n$ we obtain the constraint:

$$[[p]]^\sharp \psi \sqsupseteq \exists X_{k+1}, \dots, X_m. [[g_n]]^\sharp (\dots ([[g_1]]^\sharp \psi) \dots)$$

// m number of clause variables

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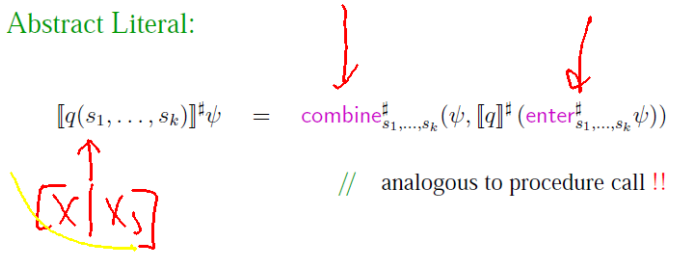
Abstract Unification:

$$[[X = t]]^\sharp \psi = \psi \wedge (X \leftrightarrow X_1 \wedge \dots \wedge X_r) \\ \text{if } \text{Vars}(t) = \{X_1, \dots, X_r\}.$$

Abstract Literal:

$$[[q(s_1, \dots, s_k)]]^\sharp \psi = \text{combine}_{s_1, \dots, s_k}^\sharp(\psi, [[q]]^\sharp(\text{enter}_{s_1, \dots, s_k}^\sharp \psi))$$

// analogous to procedure call !!



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Thereby:

$$\text{enter}_{s_1, \dots, s_k}^\sharp \psi = \text{ren}(\exists X_1, \dots, X_m. [[\bar{X}_1 = s_1, \dots, \bar{X}_k = s_k]]^\sharp \psi)$$

$$\text{combine}_{s_1, \dots, s_k}^\sharp(\psi, \psi_1) = \exists \bar{X}_1, \dots, \bar{X}_r. \psi \wedge [[\bar{X}_1 = s_1, \dots, \bar{X}_k = s_k]]^\sharp(\text{ren} \psi_1)$$

where

$$\exists X. \phi = \phi[0/X] \vee \phi[1/X]$$

$$\text{ren} \phi = \phi[\bar{X}_1/\bar{X}_1, \dots, \bar{X}_k/\bar{X}_k]$$

$$\text{ren} \phi = \phi[\bar{X}_1/X_1, \dots, \bar{X}_r/X_r]$$

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Example:

$$\begin{aligned}\text{app}(X, Y, Z) &\leftarrow X = [], Y = Z \\ \text{app}(X, Y, Z) &\leftarrow X = [H|X'], Z = [H|Z'], \text{app}(X', Y, Z')\end{aligned}$$

Then

$$\begin{aligned}[\text{app}]^\sharp(X) &\sqsupseteq X \wedge (Y \leftrightarrow Z) \\ [\text{app}]^\sharp(X) &\sqsupseteq \text{let } \psi = X \wedge H \wedge X' \wedge (Z \leftrightarrow Z') \\ &\quad \text{in } \exists H, X', Z'. \text{combine}^\sharp(\psi, [\text{app}]^\sharp(\text{enter}^\sharp(\psi)))\end{aligned}$$

where for $\psi = X \wedge H \wedge X' \wedge (Z \leftrightarrow Z')$:

$$\begin{aligned}\text{enter}^\sharp(\psi) &= X \\ \text{combine}^\sharp(\psi, X \wedge (Y \leftrightarrow Z)) &= (X \wedge H \wedge X' \wedge (Z \leftrightarrow Z') \wedge (Y \leftrightarrow Z))\end{aligned}$$

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Furthermore,

$$\begin{aligned}[\text{app}]^\sharp(Z) &\sqsupseteq X \wedge Y \wedge Z \\ [\text{app}]^\sharp(Z) &\sqsupseteq \text{let } \psi = H \wedge Z \wedge Z' \wedge (X \leftrightarrow X') \\ &\quad \text{in } \exists H, X', Z'. \text{combine}^\sharp(\psi, [\text{app}]^\sharp(\text{enter}^\sharp(\psi)))\end{aligned}$$

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Fixpoint iteration therefore yields:

$$[\text{app}]^\sharp(X) = X \wedge (Y \leftrightarrow Z) \quad [\text{app}]^\sharp(Z) = X \wedge Y \wedge Z$$

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Discussion:

- Exhaustive tabulation of the transformation $\llbracket \text{app} \rrbracket^\sharp$ is not feasible.
- Therefore, we rely on **demand-driven** fixpoint iteration !
- The evaluation starts with the evaluation of the query g , i.e., with the evaluation of $\llbracket g \rrbracket^\sharp 1$.
- The set of inspected fixpoint variables $\llbracket p \rrbracket^\sharp \psi$ yields a description of all possible calls $(:-)$
- For an efficient representation of functions $\psi \in \text{Pos}$ we rely on binary decision diagrams (BDDs).

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