Script generated by TTT

Title: Seidl: Programmoptimierung (06.02.2013)

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Remarks:

- Not all positive functions are monotonic !!!
- For k variables, there are $2^{2^k - 1} + 1$ many functions.
- The height of the complete lattice is 2^k.
- We construct an interprocedural analysis which for every predicate p determines a (monotonic) transformation

$$[\![p]\!]^\sharp : \mathsf{Pos} \to \mathsf{Pos}$$

- For every clause, $p(X_1, \ldots, X_k) \Leftarrow g_1, \ldots, g_n$ we obtain the constraint:

$$[\![p]\!]^\sharp \psi \quad \sqsupseteq \quad \exists X_{k+1}, \ldots, X_m.\, [\![g_n]\!]^\sharp (\ldots ([\![g_1]\!]^\sharp \psi) \ldots)$$

// m number of clause variables

Abstract Unification:

$$[\![X = t]\!]^\sharp \psi \quad = \quad \psi \wedge (X \leftrightarrow X_1 \wedge \ldots \wedge X_r)$$
$$\text{if} \quad Vars(t) = \{X_1, \ldots, X_r\}.$$

Abstract Literal:

$$[\![q(s_1, \ldots, s_k)]\!]^\sharp \psi \quad = \quad \mathsf{combine}^\sharp_{s_1,\ldots,s_k}(\psi, [\![q]\!]^\sharp(\mathsf{enter}^\sharp_{s_1,\ldots,s_k} \psi))$$

// analogous to procedure call !!

Thereby:

$$\mathsf{enter}^\sharp_{s_1,\ldots,s_k} \psi \quad = \quad \mathsf{ren}\,(\exists X_1, \ldots, X_m.\, [\![\bar{X}_1 = s_1, \ldots, \bar{X}_k = s_k]\!]^\sharp \psi)$$

$$\mathsf{combine}^\sharp_{s_1,\ldots,s_k}(\psi, \psi_1) \quad = \quad \exists \bar{X}_1, \ldots, \bar{X}_r.\, \psi \wedge [\![\bar{X}_1 = s_1, \ldots, \bar{X}_k = s_k]\!]^\sharp (\overline{\mathsf{ren}}\,\psi_1)$$

where

$$\exists X.\, \phi \quad = \quad \phi[0/X] \vee \phi[1/X]$$
$$\mathsf{ren}\,\phi \quad = \quad \phi[X_1/\bar{X}_1, \ldots, X_k/\bar{X}_k]$$
$$\overline{\mathsf{ren}}\,\phi \quad = \quad \phi[\bar{X}_1/X_1, \ldots, \bar{X}_r/X_r]$$

Example:

$$\mathsf{app}(X,Y,Z) \ \leftarrow\ X = [\,],\ Y = Z$$
$$\mathsf{app}(X,Y,Z) \ \leftarrow\ X = [H|X'],\ Z = [H|Z'],\ \mathsf{app}(X',Y,Z')$$

Then

$$[\![\mathsf{app}]\!]^\sharp(X) \ \sqsupseteq\ X \wedge (Y \leftrightarrow Z)$$
$$[\![\mathsf{app}]\!]^\sharp(X) \ \sqsupseteq\ \textbf{let } \psi = X \wedge H \wedge X' \wedge (Z \leftrightarrow Z') \ \textbf{in } \exists H, X', Z'.\ \mathsf{combine}^\sharp_{\dots}(\psi, [\![\mathsf{app}]\!]^\sharp(\mathsf{enter}^\sharp_{\dots}(\psi)))$$

where for $\psi = X \wedge H \wedge X' \wedge (Z \leftrightarrow Z')$:

$$\mathsf{enter}^\sharp_{\dots}(\psi) \ =\ X$$
$$\mathsf{combine}^\sharp_{\dots}(\psi, X \wedge (Y \leftrightarrow Z)) \ =\ (X \wedge H \wedge X' \wedge (Z \leftrightarrow Z') \wedge (Y \leftrightarrow Z')$$

Example (Cont.):

Furthermore,

$$[\![\mathsf{app}]\!]^\sharp(Z) \ \sqsupseteq\ X \wedge Y \wedge Z$$
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Fixpoint iteration therefore yields:

$$[\![\mathsf{app}]\!]^\sharp(X) \ =\ X \wedge (Y \leftrightarrow Z) \qquad [\![\mathsf{app}]\!]^\sharp(Z) \ =\ X \wedge Y \wedge Z$$

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$X \wedge (Z \leftrightarrow Z')$

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Discussion:

- Exhaustive tabulation of the transformation $[\![\text{app}]\!]^\sharp$ is not feasible.
- Therefore, we rely on demand-driven fixpoint iteration !
- The evaluation starts with the evaluation of the query g, i.e., with the evaluation of $[\![g]\!]^\sharp\,1$.
- The set of inspected fixpoint variables $[\![p]\!]^\sharp\,\psi$ yields a description of all possible calls :-))
- For an efficient representation of functions $\psi \in \text{Pos}$ we rely on binary decision diagrams (BDDs).

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