Script generated by TTT

Title: Seidl: Programmoptimierung (07.11.2012)

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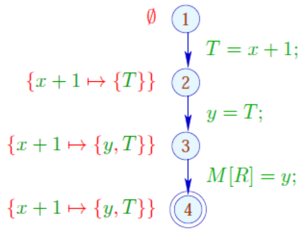
Pages: 46

In the Example:



$$[\![lab]\!] : (Expr \to 2^{Vars}) \to (Expr \to 2^{Vars})$$

$$[\![x = c;]\!]^\sharp\, V\, e' = \begin{cases} (V\, c) \cup \{x\} & \text{if } e' = c \\ (V\, e') \backslash \{x\} & \text{otherwise} \end{cases}$$

$$[\![x = y;]\!]^\sharp\, V\, e = \begin{cases} (V\, e) \cup \{x\} & \text{if } y \in V\, e \\ (V\, e) \backslash \{x\} & \text{otherwise} \end{cases}$$

$$[\![x = e;]\!]^\sharp\, V\, e' = \begin{cases} \{x\} & \text{if } e' = e \\ (V\, e') \backslash \{x\} & \text{otherwise} \end{cases}$$

$$[\![x = M[c];]\!]^\sharp\, V\, e' = (V\, e') \backslash \{x\}$$

$$[\![x = M[y];]\!]^\sharp\, V\, e' = (V\, e') \backslash \{x\}$$

$$[\![x = M[e];]\!]^\sharp\, V\, e' = \begin{cases} \emptyset & \text{if } e' = e \\ (V\, e') \backslash \{x\} & \text{otherwise} \end{cases}$$

// analogously for the diverse stores

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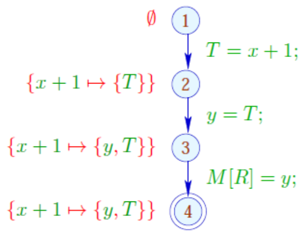

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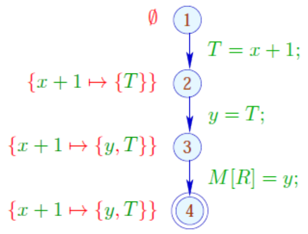


→ We propagate information in forward direction :-)

At *start*, $V_0\, e = \emptyset$ for all e;

→ $\sqsubseteq\ \subseteq \mathbb{V} \times \mathbb{V}$ is defined by:

$$V_1 \sqsubseteq V_2 \quad \text{iff} \quad V_1\, e \supseteq V_2\, e \qquad \text{for all } e$$

Observation:

The new effects of edges are distributive:

To show this, we consider the functions:

(1) $f_1^x\, V\, e = (V\, e) \backslash \{x\}$

(2) $f_2^{e,a}\, V = V \oplus \{e \mapsto a\}$

(3) $f_3^{x,y}\, V\, e = (y \in V\, e)\, ?\, (V\, e \cup \{x\}) : ((V\, e) \backslash \{x\})$

Obviously, we have:

$$\begin{array}{lll} [\![x = e;]\!]^\sharp & = & f_2^{e,\{x\}} \circ f_1^x \\ [\![x = y;]\!]^\sharp & = & f_3^{x,y} \\ [\![x = M[e];]\!]^\sharp & = & f_2^{e,\emptyset} \circ f_1^x \end{array}$$

By closure under composition, the assertion follows :-))

(1) For $f\, V\, e = (V\, e) \backslash \{x\}$, we have:

$$\begin{array}{lll} f\,(V_1 \sqcup V_2)\, e & = & ((V_1 \sqcup V_2)\, e) \backslash \{x\} \\ & = & ((V_1\, e) \cap (V_2\, e)) \backslash \{x\} \\ & = & ((V_1\, e) \backslash \{x\}) \cap ((V_2\, e) \backslash \{x\}) \\ & = & (f\, V_1\, e) \cap (f\, V_2\, e) \\ & = & (f\, V_1 \sqcup f\, V_2)\, e \qquad \text{:-)} \end{array}$$

(2) For $f\, V = V \oplus \{e \mapsto a\}$, we have:

$$\begin{array}{lll} f\,(V_1 \sqcup V_2)\, e' & = & ((V_1 \sqcup V_2) \oplus \{e \mapsto a\})\, e' \\ & = & (V_1 \sqcup V_2)\, e' \\ & = & (f\, V_1 \sqcup f\, V_2)\, e' \qquad \text{given that} \quad e \neq e' \end{array}$$

$$\begin{array}{lll} f\,(V_1 \sqcup V_2)\, e & = & ((V_1 \sqcup V_2) \oplus \{e \mapsto a\})\, e \\ & = & a \\ & = & ((V_1 \oplus \{e \mapsto a\})\, e) \cap ((V_2 \oplus \{e \mapsto a\})\, e) \\ & = & (f\, V_1 \sqcup f\, V_2)\, e \qquad \text{:-)} \end{array}$$

(3) For $f\,V\,e = (y \in V\,e)\,?\,(V\,e \cup \{x\}) : ((V\,e)\backslash\{x\})$, we have:

$$
\begin{array}{lll}
f\,(V_1 \sqcup V_2)\,e & = & (((V_1 \sqcup V_2)\,e)\backslash\{x\}) \cup (y \in (V_1 \sqcup V_2)\,e)\,?\,\{x\} : \emptyset \\
 & = & ((V_1\,e \cap V_2\,e)\backslash\{x\}) \cup (y \in (V_1\,e \cap V_2\,e))\,?\,\{x\} : \emptyset \\
 & = & ((V_1\,e \cap V_2\,e)\backslash\{x\}) \cup \\
 & & ((y \in V_1\,e)\,?\,\{x\} : \emptyset) \cap ((y \in V_2\,e)\,?\,\{x\} : \emptyset) \\
 & = & (((V_1\,e)\backslash\{x\}) \cup (y \in V_1\,e)\,?\,\{x\} : \emptyset) \cap \\
 & & (((V_2\,e)\backslash\{x\}) \cup (y \in V_2\,e)\,?\,\{x\} : \emptyset) \\
 & = & (f\,V_1 \sqcup f\,V_2)\,e \qquad :\text{-)}
\end{array}
$$

We conclude:

→ Solving the constraint system returns the MOP solution :-)

→ Let $\mathcal{V}$ denote this solution.

If $x \in \mathcal{V}[u]\,e$, then x at u contains the value of e — which we have stored in T_e

$\Longrightarrow$

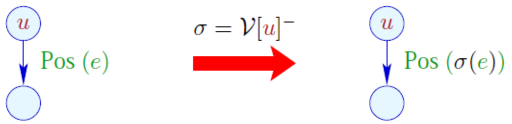
the access to x can be replaced by the access to T_e :-)

For $V \in \mathbb{V}$, let V^- denote the variable substitution with:

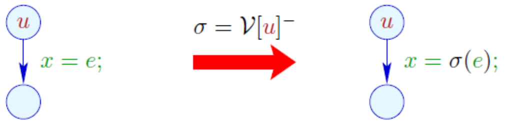
$$
V^-\,x = \begin{cases} T_e & \text{if } x \in V\,e \\ x & \text{otherwise} \end{cases}
$$

if $V\,e \cap V\,e' = \emptyset$ for $e \neq e'$. Otherwise: $V^-\,x = x$:-)

Transformation 3:



... analogously for edges with Neg (e)



Procedure as a whole:

(1) Availability of expressions: T1

+ removes arithmetic operations
− inserts superfluous moves

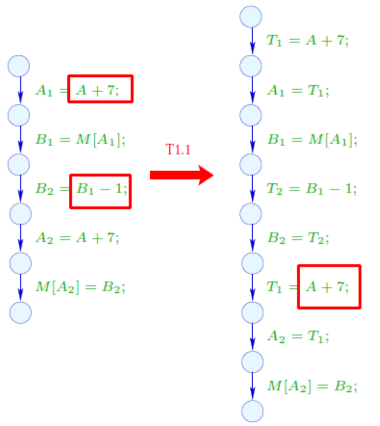
(2) Values of variables: T3

+ creates dead variables

(3) (true) liveness of variables: T2

+ removes assignments to dead variables

Example: `a[7]--;`



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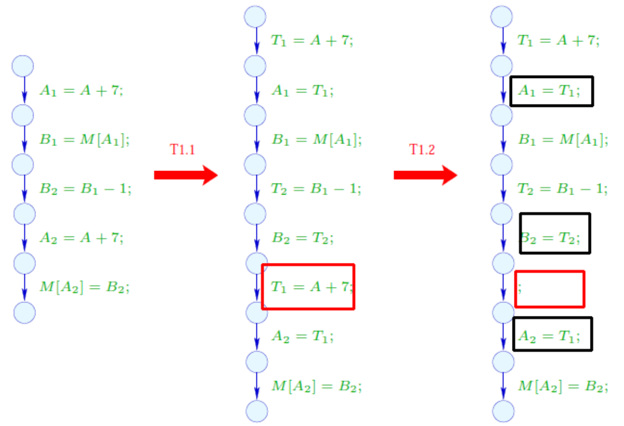
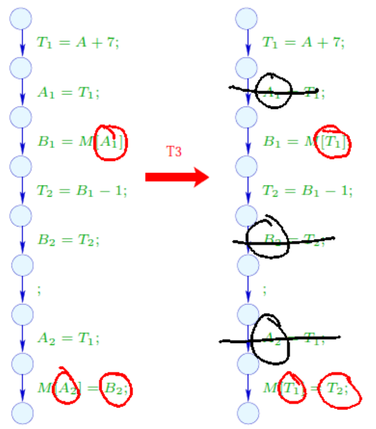


Example (cont.): `a[7]--;`



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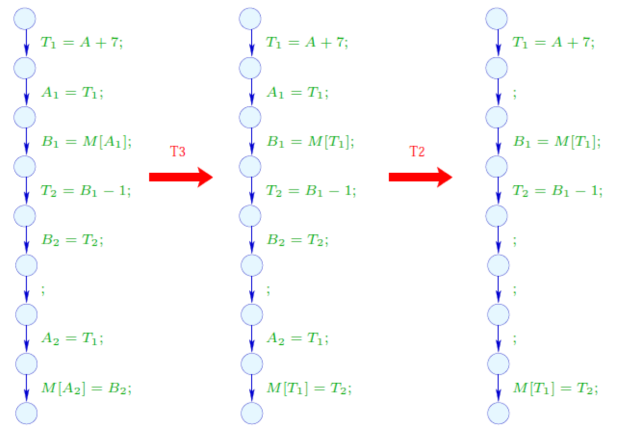


1.4 Constant Propagation

Idea:

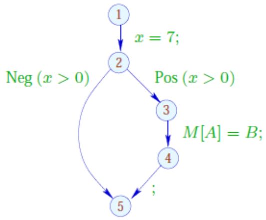
Execute as much of the code at compile-time as possible!

Example:

$x = 7;$

if $(x > 0)$

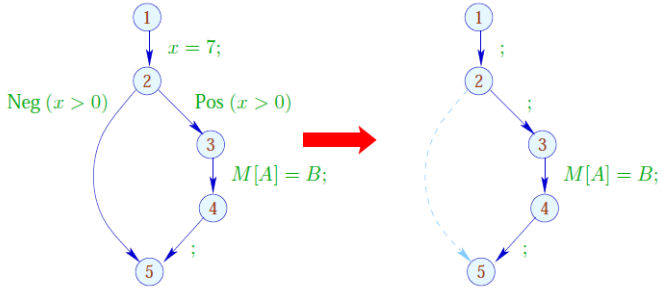
$\quad M[A] = B;$



Obviously, x has always the value 7 :-)

Thus, the memory access is always executed :-))

Goal:



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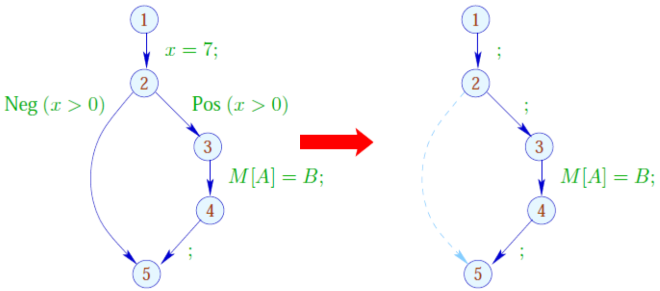

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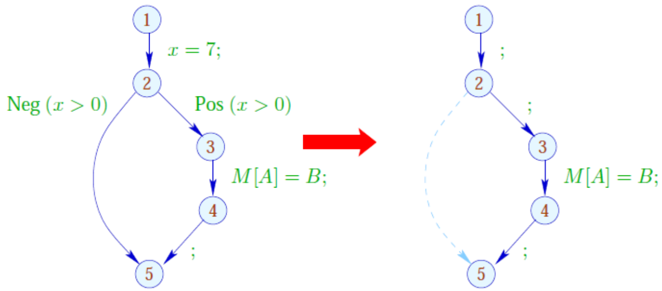

Generalization: Partial Evaluation

Neil D. Jones, DIKU, Kopenhagen

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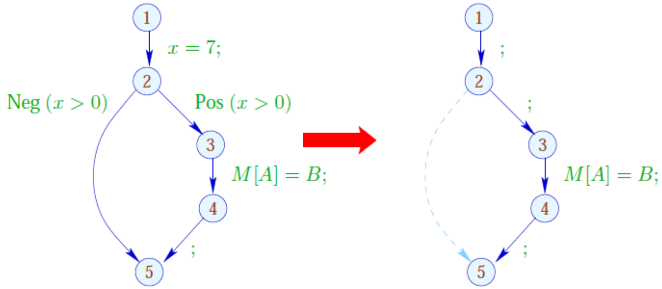

Generalization: Partial Evaluation

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Idea:

Design an analysis which for every u,

- determines the values which variables definitely have;
- tells whether u can be reached at all :-)

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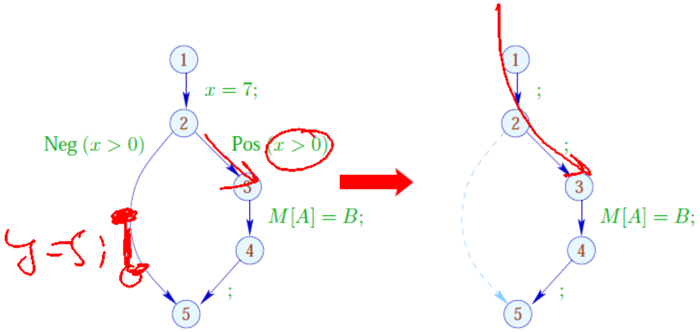


Idea:

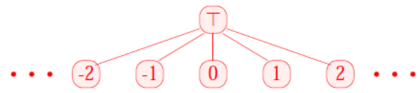
Design an analysis which for every u,

- determines the values which variables definitely have;
- tells whether u can be reached at all :-)

The complete lattice is constructed in two steps.

(1) The potential values of variables:

$$\mathbb{Z}^\top = \mathbb{Z} \cup \{\top\} \qquad \text{with} \quad x \sqsubseteq y \quad \text{iff } y = \top \text{ or } x = y$$



Caveat: $\mathbb{Z}^\top$ is not a complete lattice in itself :-(

(2) $\mathbb{D} = (Vars \to \mathbb{Z}^\top)_\perp = (Vars \to \mathbb{Z}^\top) \cup \{\perp\}$

// $\perp$ denotes: “not reachable” :-))

with $D_1 \sqsubseteq D_2$ iff $\perp = D_1$ or

$D_1\, x \sqsubseteq D_2\, x \quad (x \in Vars)$

5 5

Remark: $\mathbb{D}$ is a complete lattice :-)

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Remark: $\mathbb{D}$ is a complete lattice :-)

Consider $X \subseteq \mathbb{D}$. W.l.o.g., $\perp \not\in X$.

Then $X \subseteq Vars \to \mathbb{Z}^\top$.

If $X = \emptyset$, then $\bigsqcup X = \perp \in \mathbb{D}$:-)

If $X \neq \emptyset$, then $\bigsqcup X = D$ with

$$
\begin{array}{lll}
D\,x & = & \bigsqcup\{f\,x \mid f \in X\} \\
& = & \begin{cases} z & \text{if} \quad f\,x = z \quad (f \in X) \\ \top & \text{otherwise} \end{cases}
\end{array}
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:-))

$$\{x \mapsto 5, y \mapsto 2\} \sqcup \{x \mapsto 7, y \mapsto 2\} \sqcup \bot = \{x \mapsto \top, y \mapsto 2\}$$

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\end{array}
$$

:-))

For every edge $k = (\_, lab, \_)$, construct an effect function $[\![k]\!]^\sharp = [\![lab]\!]^\sharp : \mathbb{D} \to \mathbb{D}$ which simulates the concrete computation.

Obviously, $[\![lab]\!]^\sharp\,\bot = \bot$ for all lab :-)

Now let $\bot \neq D \in Vars \to \mathbb{Z}^\top$.

Idea:

- We use D to determine the values of expressions.

$lab \equiv x = x + 1;$

$D = \{x \mapsto 4, y \mapsto 3\}$

$[\![lab]\!]^\# D = \{x \mapsto 5, y \mapsto 3\}$

Idea:

- We use D to determine the values of expressions.
- For some sub-expressions, we obtain $\top$:-)

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  $\Longrightarrow$

  We must replace the concrete operators $\Box$ by abstract operators $\Box^\sharp$ which can handle $\top$:

$$a \,\Box^\sharp\, b = \begin{cases} \top & \text{if} \quad a = \top \text{ or } b = \top \\ a \,\Box\, b & \text{otherwise} \end{cases}$$

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- The abstract operators allow to define an abstract evaluation of expressions:

$$[\![e]\!]^\sharp \;:\; (Vars \to \mathbb{Z}^\top) \to \mathbb{Z}^\top$$

Abstract evaluation of expressions is like the concrete evaluation — but with abstract values and operators. Here:

$$\begin{array}{lll} [\![c]\!]^\sharp\, D & = & c \\ [\![e_1 \,\Box\, e_2]\!]^\sharp\, D & = & [\![e_1]\!]^\sharp\, D \,\Box^\sharp\, [\![e_2]\!]^\sharp\, D \end{array}$$

... analogously for unary operators :-)

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... analogously for unary operators :-)

Example: $D = \{x \mapsto 2, y \mapsto \top\}$

$$\begin{array}{lll} [\![x+7]\!]^\sharp\, D & = & [\![x]\!]^\sharp\, D +^\sharp [\![7]\!]^\sharp\, D \\ & = & 2 +^\sharp 7 \\ & = & 9 \\ [\![x-y]\!]^\sharp\, D & = & 2 -^\sharp \top \\ & = & \top \end{array}$$

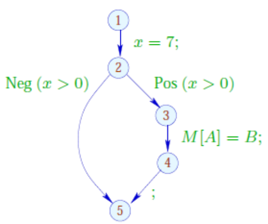
Thus, we obtain the following effects of edges $[\![lab]\!]^\sharp$:

$$\begin{array}{lll} [\![;]\!]^\sharp\, D & = & D \\ [\![\mathrm{Pos}\,(e)]\!]^\sharp\, D & = & \begin{cases} \bot & \text{if} \quad 0 = [\![e]\!]^\sharp\, D \\ D & \text{otherwise} \end{cases} \\ [\![\mathrm{Neg}\,(e)]\!]^\sharp\, D & = & \begin{cases} D & \text{if} \quad 0 \sqsubseteq [\![e]\!]^\sharp\, D \\ \bot & \text{otherwise} \end{cases} \\ [\![x = e;]\!]^\sharp\, D & = & D \oplus \{x \mapsto [\![e]\!]^\sharp\, D\} \\ [\![x = M[e];]\!]^\sharp\, D & = & D \oplus \{x \mapsto \top\} \\ [\![M[e_1] = e_2;]\!]^\sharp\, D & = & D \end{array}$$

... whenever $D \neq \bot$:-)

At *start*, we have $D_\top = \{x \mapsto \top \mid x \in Vars\}$.

Example:



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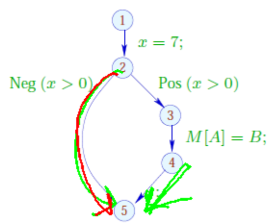

1	$\{x \mapsto \top\}$
2	$\{x \mapsto 7\}$
3	$\{x \mapsto 7\}$
4	$\{x \mapsto 7\}$
5	$\bot \sqcup \{x \mapsto 7\} = \{x \mapsto 7\}$

$[\![\mathrm{Neg}(x>0)]\!]^\#(\{x \mapsto 7\}) = \bot$

The abstract effects of edges $[\![k]\!]^\sharp$ are again composed to the effects of paths $\pi = k_1 \ldots k_r$ by:

$$[\![\pi]\!]^\sharp = [\![k_r]\!]^\sharp \circ \ldots \circ [\![k_1]\!]^\sharp \quad : \mathbb{D} \to \mathbb{D}$$

Idea for Correctness: Abstract Interpretation

Cousot, Cousot 1977

Patrick Cousot, ENS, Paris

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Cousot, Cousot 1977

Establish a description relation Δ between theconcrete values and their descriptions with:

$$x \,\Delta\, a_1 \;\wedge\; a_1 \sqsubseteq a_2 \;\Longrightarrow\; x \,\Delta\, a_2$$

Concretization: $\gamma\, a = \{x \mid x \,\Delta\, a\}$

// returns the set of described values :-)

$\gamma(2) = 2$ $\gamma(\top) = \mathbb{Z}$

(1) Values: $\Delta \subseteq \mathbb{Z} \times \mathbb{Z}^\top$

$$z \,\Delta\, a \quad \text{iff} \quad z = a \vee a = \top$$

Concretization:

$$\gamma\, a = \begin{cases} \{a\} & \text{if} \quad a \sqsubset \top \\ \mathbb{Z} & \text{if} \quad a = \top \end{cases}$$

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(2) Variable Assignments: $\Delta \subseteq (Vars \to \mathbb{Z}) \times (Vars \to \mathbb{Z}^\top)_\perp$

$$\rho \,\Delta\, D \quad \text{iff} \quad D \neq \perp \;\wedge\; \rho\, x \,\Delta\, D\, x \quad (x \in Vars)$$

Concretization:

$$\gamma\, D = \begin{cases} \emptyset & \text{if} \quad D = \perp \\ \{\rho \mid \forall x : (\rho\, x) \,\Delta\, (D\, x)\} & \text{otherwise} \end{cases}$$