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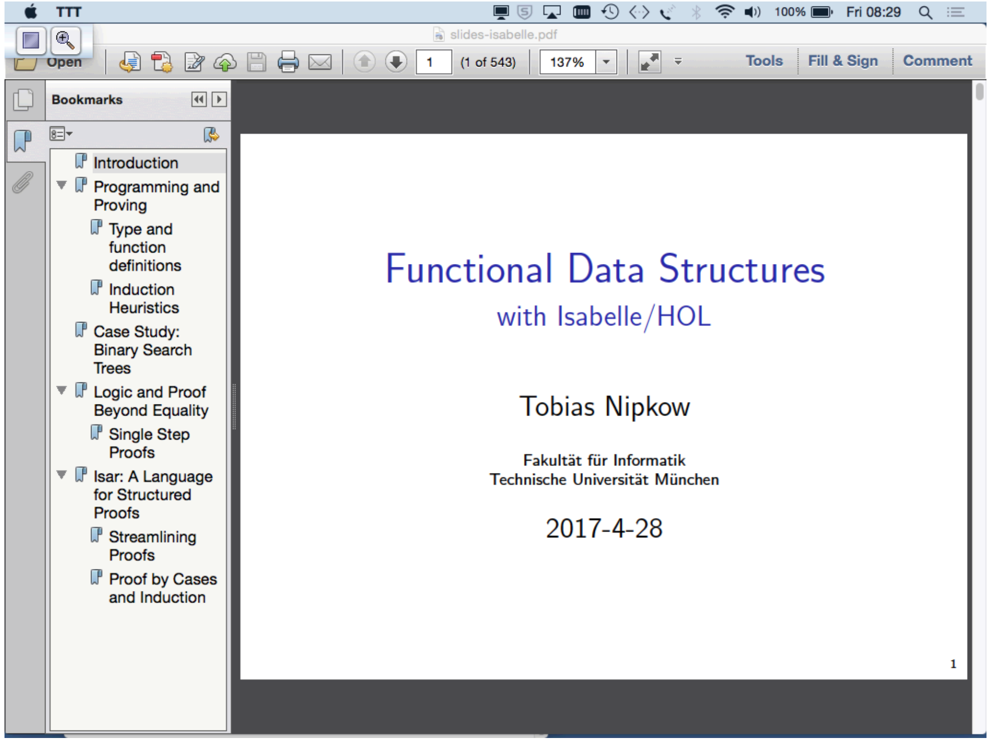
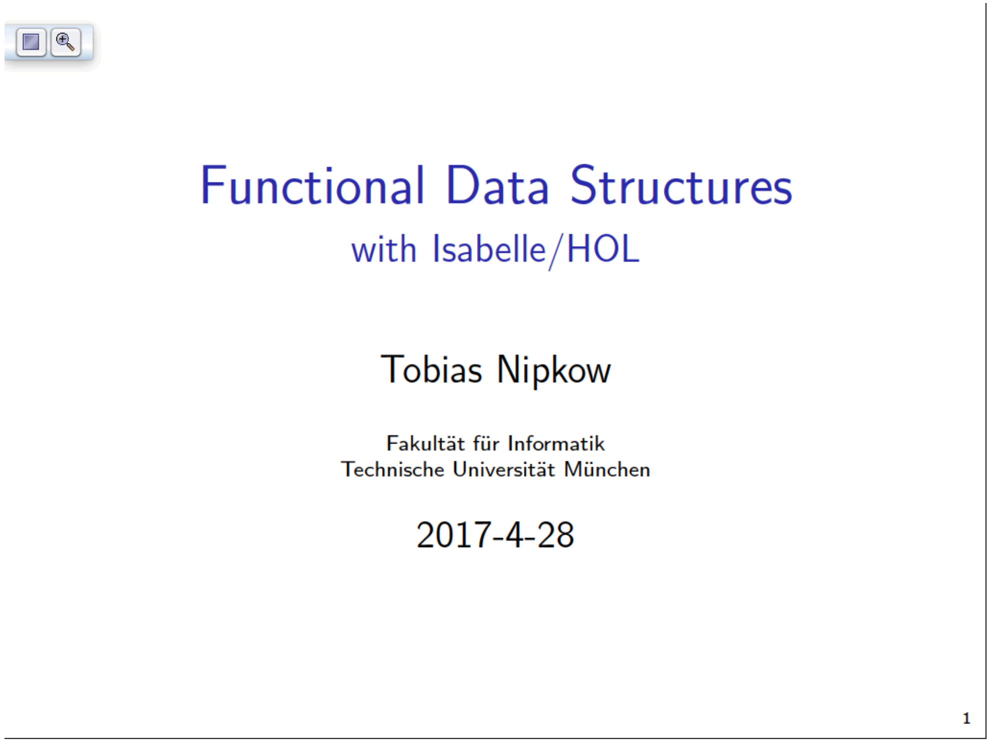


Functional Data Structures
with Isabelle/HOL

Tobias Nipkow

Fakultät für Informatik
Technische Universität München

2017-4-28



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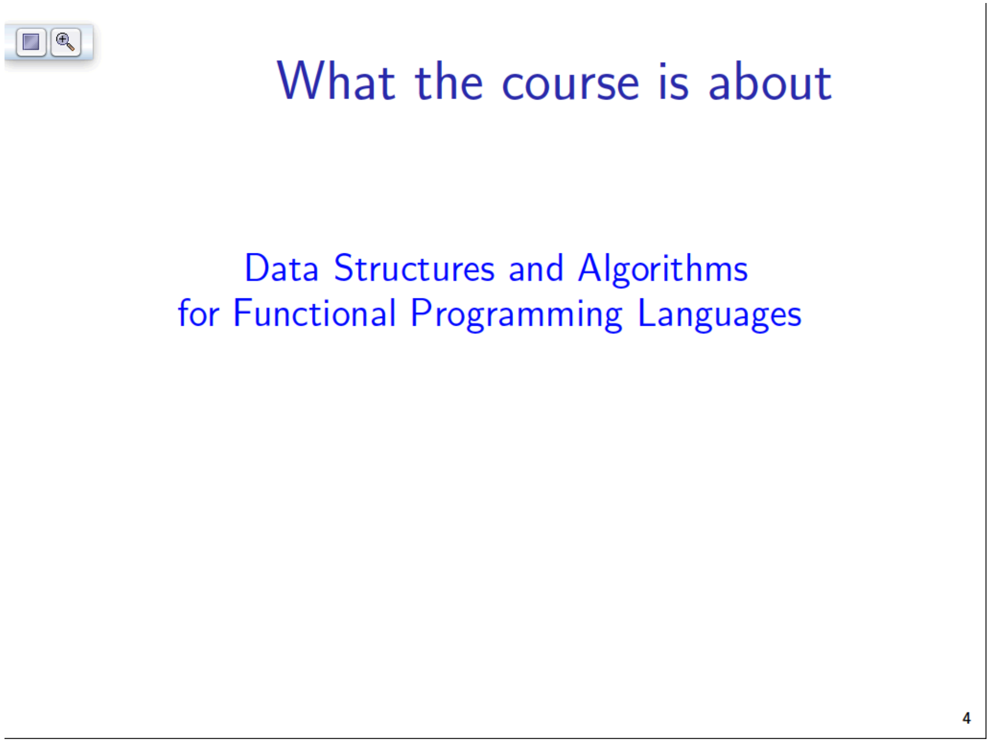

What the course is about

Data Structures and Algorithms
for Functional Programming Languages

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Data Structures and Algorithms
for Functional Programming Languages

The code is not enough!

Formal Correctness and Complexity Proofs
with the Proof Assistant *Isabelle*

Proof Assistants

- You give the structure of the proof
- The PA checks the correctness of each step

Terminology

Formal = machine-checked
Verification = formal correctness proof

Two landmark verifications

C compiler

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C compiler
Competitive with `gcc -O1`

Xavier Leroy
INRIA Paris
using Coq

Two landmark verifications

C compiler
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Operating system
microkernel (L4)

Gerwin Klein (& Co)
NICTA Sydney
using Isabelle

Overview of course

- Week 1–5: Introduction to Isabelle
- Rest of semester: Search trees, priority queues, etc and their (amortized) complexity

---

What we expect from you

Functional programming experience with an ML/Haskell-like language

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First course in data structures and algorithms

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First course in discrete mathematics

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Functional programming experience with an ML/Haskell-like language

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You will not survive this course without doing the time-consuming homework

---

Part I

Isabelle

Quiz

Which of the following formulas have the same meaning?

1. $A \Longrightarrow (B \Longrightarrow C)$
2. $(A \Longrightarrow B) \Longrightarrow C$
3. $(A \wedge B) \Longrightarrow C$

Notation

Implication associates to the right:

$$A \Longrightarrow B \Longrightarrow C \quad \text{means} \quad A \Longrightarrow (B \Longrightarrow C)$$

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Implication associates to the right:

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Similarly for other arrows: $\Rightarrow$, $\longrightarrow$

$$\frac{A_1 \quad \dots \quad A_n}{B} \quad \text{means} \quad A_1 \Longrightarrow \dots \Longrightarrow A_n \Longrightarrow B$$

---

1. Overview of Isabelle/HOL
2. Type and function definitions
3. Induction Heuristics
4. Simplification

HOL = Higher-Order Logic
HOL = Functional Programming + Logic

HOL has

- datatypes
- recursive functions
- logical operators

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- For the moment: only $term = term$

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HOL = Functional Programming + Logic

HOL has

- datatypes
- recursive functions
- logical operators

HOL is a programming language!

Higher-order = functions are values, too!

HOL Formulas:

- For the moment: only $term = term$, e.g. $1 + 2 = 4$
- Later: $\wedge$, $\vee$, $\longrightarrow$, $\forall$, $\ldots$

---

1. Overview of Isabelle/HOL
   - Types and terms
   - Interface
   - By example: types $bool$, nat and $list$
   - Numeric Types
   - Summary

Types

Basic syntax:

$$\begin{array}{lcll} \tau & ::= & (\tau) & \\ & | & bool \mid nat \mid int \mid \ldots & \text{base types} \end{array}$$

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 & \mid & \ldots & \text{user-defined types}
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λ-calculus

Terms must be well-typed

(the argument of every function call must be of the right type)

Notation:
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$$\frac{t :: \tau_1 \Rightarrow \tau_2 \qquad u :: \tau_1}{t\ u :: \tau_2}$$

Type inference

Isabelle automatically computes the type of each variable in a term.

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User can help with *type annotations* inside the term.
Example: $f\ (x{::}nat)$

Currying

Thou shalt Curry your functions

- Curried: $f :: \tau_1 \Rightarrow \tau_2 \Rightarrow \tau$
- Tupled: $f' :: \tau_1 \times \tau_2 \Rightarrow \tau$

Predefined syntactic sugar

- *Infix:* $+$, $-$, $*$, $\#$, $@$, ...

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Prefix binds more strongly than infix:

$$! \quad f\,x + y \ \equiv\ (f\,x) + y \ \not\equiv\ f\,(x + y) \quad !$$

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Syntax: `theory` $MyTh$
`imports` $T_1 \ldots T_n$
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(definitions, theorems, proofs, ...)*
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In `.thy` files:
Types, terms and formulas need to be inclosed in `"`

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Concrete syntax

In `.thy` files:
Types, terms and formulas need to be inclosed in "

Except for single identifiers

" normally not shown on slides

---

isabelle jedit

- Based on *jEdit* editor
- Processes Isabelle text automatically when editing `.thy` files

---

`Overview_Demo.thy`

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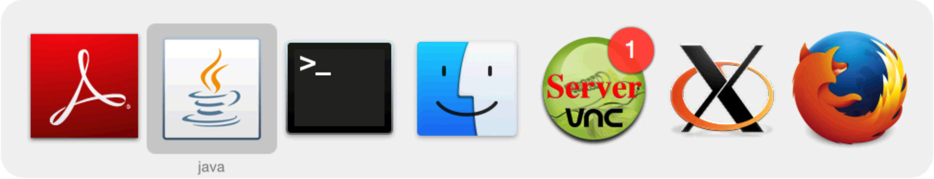

Type *bool*

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A *formula* is a term of type *bool*

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datatype $bool = True \mid False$

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if-and-only-if: $=$

Type nat

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$0,1,2,\ldots :: 'a, \quad + :: \ 'a \Rightarrow 'a \Rightarrow 'a$

You need type annotations: $1 :: nat,\ x + (y{::}nat)$
unless the context is unambiguous: $Suc\ z$

`Nat_Demo.thy`

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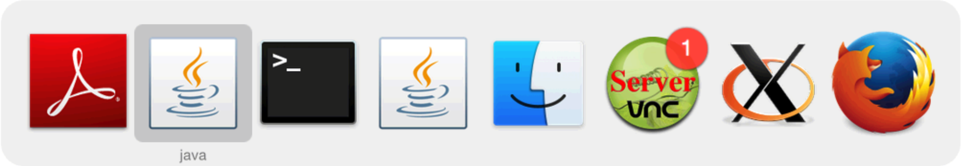

An informal proof

Lemma $add\ m\ 0 = m$

Proof by induction on m.

- Case 0 (the base case):
  $add\ 0\ 0 = 0$ holds by definition of add.
- Case $Suc\ m$ (the induction step):
  We assume $add\ m\ 0 = m$,
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  We need to show $add\ (Suc\ m)\ 0 = Suc\ m$.
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  $$\begin{aligned} add\ (Suc\ m)\ 0 &= Suc\ (add\ m\ 0) && \text{by def. of } add \\ &= Suc\ m && \text{by IH} \end{aligned}$$

Type $'a\ list$

Lists of elements of type $'a$

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Some lists: Nil,

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Lists of elements of type $'a$

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Some lists: Nil, $Cons\ 1\ Nil$, $Cons\ 1\ (Cons\ 2\ Nil)$, $\ldots$

Syntactic sugar:

- $[] = Nil$: empty list
- $x\ \#\ xs = Cons\ x\ xs$:
  list with first element x (*"head"*) and rest xs (*"tail"*)
- $[x_1, \ldots, x_n] = x_1\ \#\ \ldots\ x_n\ \#\ []$

Structural Induction for lists

To prove that $P(xs)$ for all lists xs, prove

- $P([])$ and
- for arbitrary but fixed x and xs,
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$$\frac{P([]) \qquad \bigwedge x\ xs.\ P(xs) \Longrightarrow P(x\#xs)}{P(xs)}$$

`List_Demo.thy`

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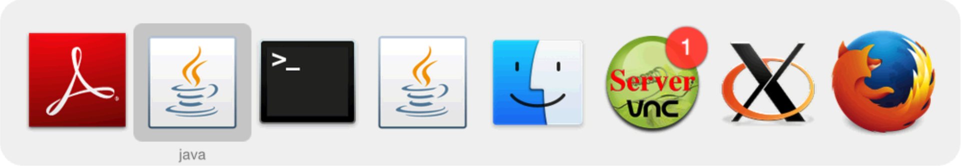

Large library: `HOL/List.thy`

Included in `Main`.

Don't reinvent, reuse!

---

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   - Types and terms
   - Interface
   - By example: types *bool*, *nat* and *list*
   - Numeric Types
   - Summary

Numeric types: *nat*, *int*, *real*

Need conversion functions (inclusions):

$$\begin{aligned} int &:: nat \Rightarrow int \\ real &:: nat \Rightarrow real \\ real\_of\_int &:: int \Rightarrow real \end{aligned}$$

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Examples

$(i::\mathit{int}) + (n::\mathit{nat})$

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$((n::\mathit{nat}) + n) :: \mathit{real}$

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Examples

$(i::\mathit{int}) + (n::\mathit{nat}) \rightsquigarrow i + \mathit{int}\ n$

$((n::\mathit{nat}) + n) :: \mathit{real} \rightsquigarrow \mathit{real}(n+n)$, $\mathit{real}\ n + \mathit{real}\ n$

Numeric types: *nat*, *int*, *real*

Coercion in the other direction:

$$nat \quad :: \quad int \Rightarrow nat$$

Overloaded arithmetic operations

- Basic arithmetic functions are overloaded:
  $+,\ -,\ * :: 'a \Rightarrow 'a \Rightarrow 'a$
  $- :: 'a \Rightarrow 'a$
- Division on *nat* and *int*:
  $div,\ mod :: 'a \Rightarrow 'a \Rightarrow 'a$

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- Absolute value: $abs :: 'a \Rightarrow 'a$

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- **datatype** defines (possibly) recursive data types.

- **fun** defines (possibly) recursive functions by pattern-matching over datatype constructors.

Proof methods

- *induction* performs structural induction on some variable (if the type of the variable is a datatype).

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“=” is used only from left to right!

Proofs

General schema:

lemma *name*: `"..."`
apply (...)
apply (...)
⋮
done

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General schema:

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apply `(...)`
apply `(...)`
⋮
done

If the lemma is suitable as a simplification rule:

lemma *name*`[simp]:   "..."`

Top down proofs

Command

sorry

“completes” any proof.

The proof state

$$1.\ \bigwedge x_1 \dots x_p.\ A \Longrightarrow B$$

Multiple assumptions

$$[\![\ A_1; \dots ; A_n\ ]\!] \Longrightarrow B$$

abbreviates

$$A_1 \Longrightarrow \dots \Longrightarrow A_n \Longrightarrow B$$

Multiple assumptions

$$[\![\, A_1; \ldots ; A_n \,]\!] \Longrightarrow B$$

abbreviates

$$A_1 \Longrightarrow \ldots \Longrightarrow A_n \Longrightarrow B$$

$;$ ≈ “and”

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