## Script generated by TTT

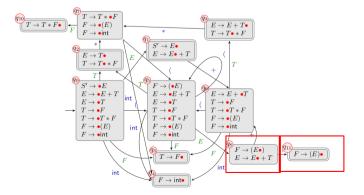
Title: Petter: Compilerbau (07.06.2018)

Date: Thu Jun 07 14:18:55 CEST 2018

Duration: 88:49 min

Pages: 24

# Canonical LR(0)-Automaton



#### LR(0)-Parser

... for example:

$$\begin{array}{lll} q_1 & = & \{[S' \to E \bullet], & & \\ & [E \to E \bullet + T]\} & & & \\ q_2 & = & \{[E \to T \bullet], & & q_9 & = & \{[E \to E + T \bullet], \\ & [T \to T \bullet *F]\} & & [T \to T \bullet *F]\} \\ q_3 & = & \{[T \to F \bullet]\} & q_{10} & = & \{[T \to T * F \bullet]\} \\ q_4 & = & \{[F \to \mathsf{int} \bullet]\} & q_{11} & = & \{[F \to (E) \bullet]\} \end{array}$$

The final states  $q_1, q_2, q_9$  contain more than one admissible item  $\Rightarrow$  non deterministic!

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#### LR(0)-Parser

#### Idea for a parser:

- The parser manages a viable prefix  $\alpha = X_1 \dots X_m$  on the pushdown and uses LR(G), to identify reduction spots.
- It can reduce with  $A \to \gamma$  , if  $[A \to \gamma \bullet]$  is admissible for  $\alpha$

#### Optimization:

We push the states instead of the  $X_i$  in order not to process the pushdown's content with the automaton anew all the time. Reduction with  $A \to \gamma$  leads to popping the uppermost  $|\gamma|$  states and continue with the state on top of the stack and input A.

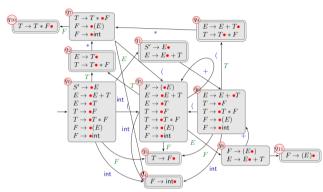
#### Attention:

This parser is only deterministic, if each final state of the canonical LR(0)-automaton is conflict free.

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# Canonical LR(0)-Automaton





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#### LR(0)-Parser

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$$\begin{array}{lll} q_1 & = & \{[T \to T *F \bullet]\} \end{array}$$
 
$$\begin{array}{lll} q_{10} & = & \{[T \to T *F \bullet]\} \end{array}$$
 
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This parser is only deterministic, if each final state of the canonical LR(0)-automaton is conflict free.

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# LR(0)-Parser

# The construction of the LR(0)-parser:

#### **Transitions:**

```
Shift: (p,a,p\,q) if q=\delta(p,a)\neq\emptyset Reduce: (p\,q_1\ldots q_m,\epsilon,p\,q) if [A\to X_1\ldots X_m\,ullet]\in q_m, q=\delta(p,A) Finish: (q_0\,p,\epsilon,f) if [S'\to Sullet]\in p with LR(G)=(Q,T,\delta,q_0,F).
```

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# LR(0)-Parser

#### Attention:

Unfortunately, the LR(0)-parser is in general non-deterministic.

We identify two reasons:



Shift-Reduce-Conflict:

$$\begin{bmatrix} A \to \gamma \bullet \end{bmatrix}, \begin{bmatrix} [A' \to \alpha \bullet a \beta] \end{bmatrix} \in \mathbf{q} \quad \text{with} \quad \boxed{a \in T}$$
 for a state  $\mathbf{q} \in Q$ .

Those states are called LR(0)-unsuited.

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# LR(k)-Grammars

Idea: Consider k-lookahead in conflict situations.

#### Definition:

The reduced contextfree grammar G is called LR(k)-grammar, if for  $\mathsf{First}_{|\alpha\beta|+k}(\alpha\beta w) = \mathsf{First}_{|\alpha\beta|+k}(\alpha'\beta'w')$  with:

$$\left. \begin{array}{cccc}
S & \rightarrow_R^* & \alpha \, A \, w & \rightarrow & \alpha \, \beta \, w \\
S & \rightarrow_R^* & \alpha' \, A' \, w' & \rightarrow & \alpha' \, \beta' \, w'
\end{array} \right\} \text{ follows: } \alpha = \alpha' \, \land \, \beta = \beta' \, \land \, A = A'$$

# Revisiting the Conflicts of the LR(0)-Automaton

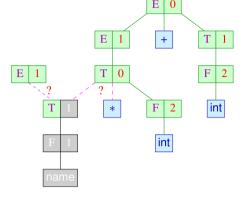
What differenciates the particular Reductions and Shifts?

Input:

$$*2 + 40$$

Pushdown:

$$(q_0 T)$$



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# LR(k)-Grammars

for example:

(1) 
$$S \rightarrow A \mid B$$
  $A \rightarrow a A b \mid 0$   $B \rightarrow a B b b \mid 1$ 

## LR(1)-Parsing

Idea: Let's equip items with 1-lookahead

# Definition LR(1)-Item

An 
$$LR(1)$$
-item is a pair  $[B \rightarrow \alpha \bullet \beta, x]$  with

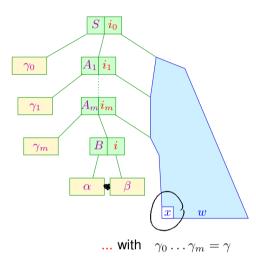
$$x \in \mathsf{Follow}_1(B) = \bigcup \{\mathsf{First}_1(\nu) \mid S \to^* \mu \, B \, \nu \}$$

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#### Admissible LR(1)-Items

The item  $[B \to \alpha \bullet \beta(x)]$  is admissable for  $\gamma \alpha$  if:

$$S \to_R^* \gamma B w$$
 with  $\{x\} = \mathsf{First}_1(w)$ 



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# The Characteristic LR(1)-Automaton

The set of admissible LR(1)-items for viable prefixes is again computed with the help of the finite automaton c(G, 1).

The automaton c(G,1):

States: LR(1)-items

Start state:  $[S' \rightarrow \bullet S, \epsilon]$ 

Final states:  $\{[B \to \gamma \bullet, x] \mid B \to \gamma \in P, x \in \mathsf{Follow}_1(B)\}$ 

**Transitions:** 

(1)  $([A \to \alpha \bullet X \beta, x], X, [A \to \alpha X \bullet \beta, x]), X \in (N \cup T)$ (2)  $([A \to \alpha \bullet B \beta, x], x, [B \to \bullet \gamma, x']), A \to \alpha B \beta, B \to \gamma \in P, x' \in \mathsf{First}_1(\beta) \odot_1 \{x\}$ 

# The Canonical LR(1)-Automaton

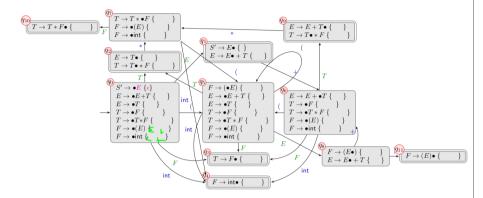
The canonical LR(1)-automaton LR(G,1) is created from c(G,1), by performing arbitrarily many  $\epsilon$ -transitions and then making the resulting automaton deterministic ...



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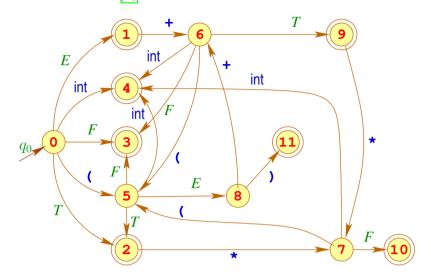
# Canonical LR(1)-Automaton

For example:



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# The Canonical LR(1)-Automaton

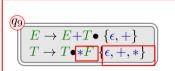


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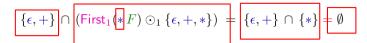
# The Canonical LR(1)-Automaton

## Discussion:

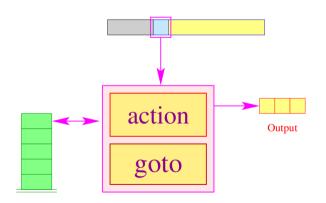
- In the example, the number of states was almost doubled ... and it can become even worse
- The conflicts in states  $q_1, q_2, q_9$  are now resolved ! e.g. we have:



with:



# The LR(1)-Parser:



• The goto-table encodes the transitions:

$$goto[q, X] = \delta(q, X) \in Q$$

ullet The action-table describes for every state q and possible lookahead w the necessary action.

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# The LR(1)-Parser:

# The construction of the LR(1)-parser:

```
States: Q \cup \{f\} (f fresh)
```

Start state:  $q_0$ Final state: fTransitions:

Shift:  $(p,a,p\,q) \quad \text{if} \quad q = \operatorname{goto}[q,a], \\ s = \operatorname{action}[p,w]$ 

**Reduce:**  $(p q_1 \dots q_{|\beta|}, \epsilon, p q)$  if  $[A \to \beta \bullet] \in q_{|\beta|}, q = \text{goto}(p, A)$ .

 $[A \to \beta \bullet] = \operatorname{action}[q_{|\beta|}, w]$ 

Finish:  $(q_0 p, \epsilon, f)$  if  $[S' \to S \bullet] \in p$ 

with  $LR(G,1) = (Q,T,\delta,q_0,F)$ .

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# The LR(1)-Parser:

Possible actions are:

**shift** // Shift-operation  $reduce(A \rightarrow \gamma)$  // Reduction with

Reduction with callback/output Error

error //

... for example:

action	\$	int	(	)	+	*
$q_1$	S', <b>0</b>				S	
$q_2$	E, <b>1</b>				E, <b>1</b>	S
$q_2'$				E, <b>1</b>	E, <b>1</b>	S
$q_3$	T, <b>1</b>				T, <b>1</b>	T, 1
$q_3'$				T, <b>1</b>	T, <b>1</b>	T, 1
$q_4$	F, <b>1</b>				F, <b>1</b>	F, 1
$q_4'$				F, <b>1</b>	F, <b>1</b>	F, 1
$q_9$	E, 0				$E, {\color{red}0}$	S
$q_9'$				$E, {\color{red}0}$	$E, {\color{red}0}$	S
$q_{10}$	T, 0				$T, {\color{red}0}$	T, 0
$q_{10}'$				$T, {\color{red}0}$	$T, {\color{red}0}$	T, 0
$q_{11}$	F, 0				$F, {\color{red}0}$	$F, {\color{red}0}$
$q_{11}'$				$F, {\color{red}0}$	$F, {\color{red}0}$	F, 0

## The Canonical LR(1)-Automaton

In general: We identify two conflicts:

**Reduce-Reduce-Conflict:** 

$$[A \to \gamma \bullet, x], [A' \to \gamma' \bullet, x] \in q \text{ with } A \neq A' \lor \gamma \neq \gamma'$$

**Shift-Reduce-Conflict:** 

for a state  $q \in Q$ .

Such states are now called LR(1)-unsuited

# **Precedences**

Many parser generators give the chance to fix Shift-/Reduce-Conflicts by patching the action table either by hand or with *token precedences*.

... for example:

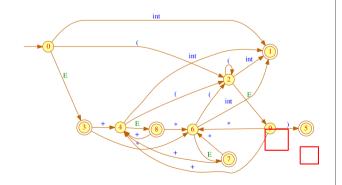
$$S' \rightarrow E^{0}$$

$$E \rightarrow E + E^{0}$$

$$\mid E * E^{1}$$

$$\mid (E)^{2}$$

$$\mid \text{int}^{3}$$



# What if precedences are not enough?

Example (very simplified lambda expressions):

```
\begin{array}{ccc} E & \rightarrow & (E)^{0} | \operatorname{ident}^{1} | L^{2} \\ L & \rightarrow & \langle \operatorname{args} \rangle \Rightarrow E^{0} \\ \langle \operatorname{args} \rangle & \rightarrow & (\langle \operatorname{idlist} \rangle)^{0} | \operatorname{ident}^{1} \\ \langle \operatorname{idlist} \rangle & \rightarrow & \langle \operatorname{idlist} \rangle \operatorname{ident}^{0} | \operatorname{ident}^{1} \end{array}
```

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