Script generated by TTT

Title: Petter: Compilerbau (27.04.2015)

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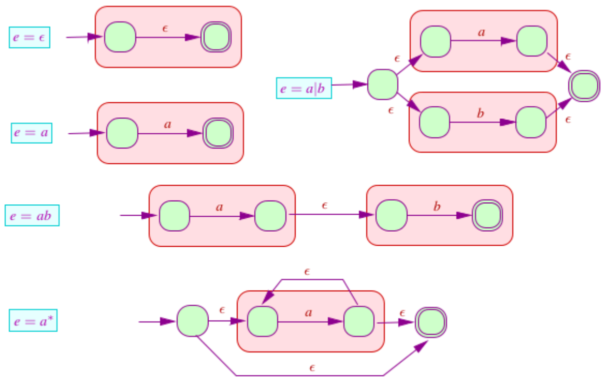
Pages: 63

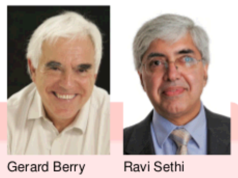
In linear time from Regular Expressions to NFAs



Thompson's Algorithm

Produces $\mathcal{O}(n)$ states for regular expressions of length n.

Berry-Sethi Approach



Berry-Sethi Algorithm

Produces exactly $n + 1$ states without ϵ-transitions and demonstrates → *Equality Systems* and → *Attribute Grammars*

Idea:

The automaton tracks (conceptionally via a marker "•"), in the syntax tree of a regular expression, which subexpressions in e are reachable consuming the rest of input w.

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Glushkov Algorithm

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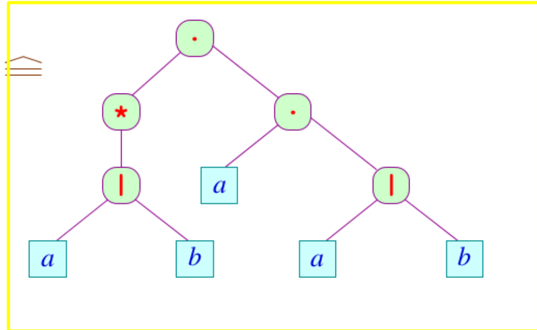
Berry-Sethi Approach

... for example:

$(a|b)^*a(a|b)$ $\widehat{=}$



Berry-Sethi Approach

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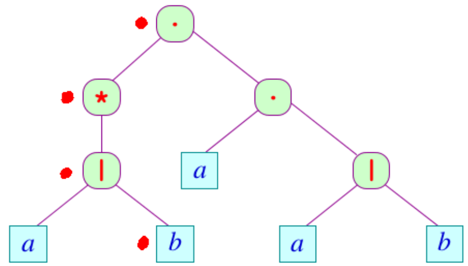
$w = bbaa$:



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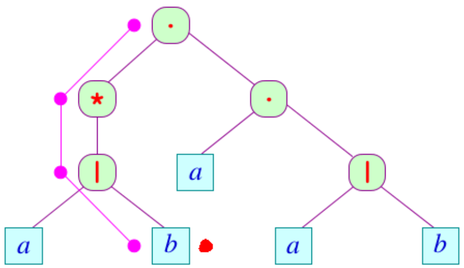

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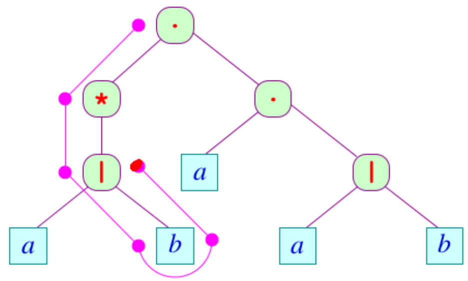

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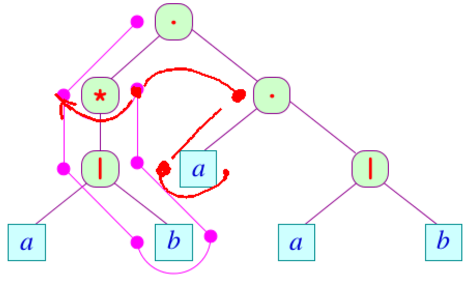

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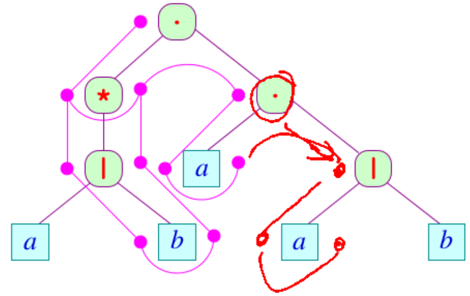

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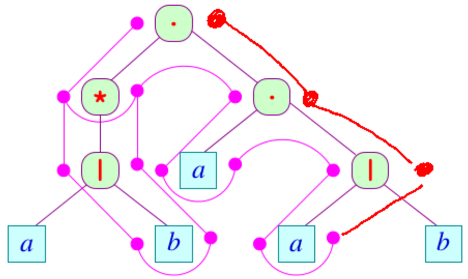

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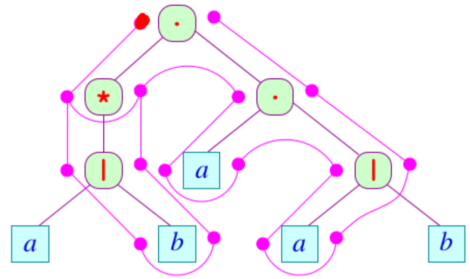

Berry-Sethi Approach

In general:

- Input is only consumed at the leaves.
- Navigating the tree does not consume input $\rightarrow$ ϵ-transitions
- For a formal construction we need identifiers for states.
- For a node n's identifier we take the subexpression, corresponding to the subtree dominated by n.
- There are possibly identical subexpressions in one regular expression.

$\Longrightarrow$ we enumerate the leaves ...

Berry-Sethi Approach

a|b

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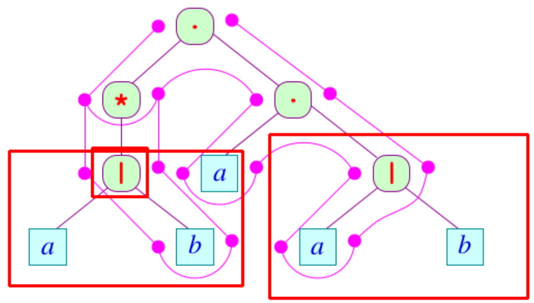

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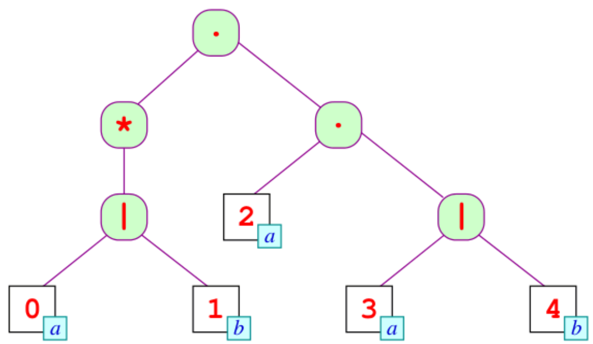

Berry-Sethi Approach (naive version)

Construction (naive version):

States: $\bullet r$, $r\bullet$ with r nodes of e;
Start state: $\bullet e$;
Final state: $e\bullet$;
Transitions: for leaves $r \equiv \boxed{i \mid x}$ we require: $(\bullet r, x, r\bullet)$.

The leftover transitions are:

r	Transitions
$r_1 \mid r_2$	$(\bullet r, \epsilon, \bullet r_1)$
	$(\bullet r, \epsilon, \bullet r_2)$
	$(r_1\bullet, \epsilon, r\bullet)$
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$r_1 \cdot r_2$	$(\bullet r, \epsilon, \bullet r_1)$
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Discussion:

- Most transitions navigate through the expression
- The resulting automaton is in general nondeterministic

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Idea:

Pre-compute helper attributes during D(epth)F(irst)S(earch)!

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- Most transitions navigate through the expression
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⇒ Strategy for the sophisticated version:
Avoid generating ϵ-transitions

Idea:

Pre-compute helper attributes during D(epth)F(irst)S(earch)!

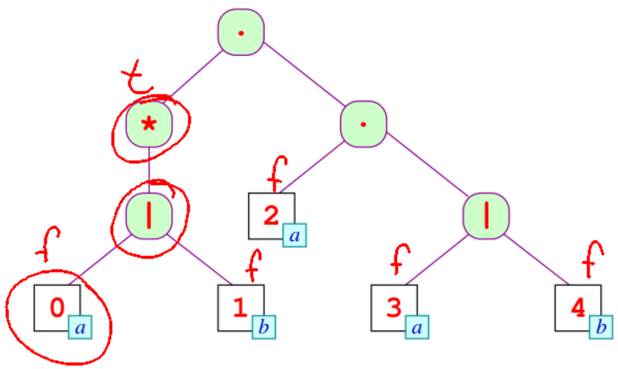
Necessary node-attributes:

- **first** the set of read states below r, which may be reached first, when descending into r.
- **next** the set of read states to the right of r, which may be reached first in the traversal after r.
- **last** the set of read states below r, which may be reached last when descending into r.
- **empty** can the subexpression r consume ϵ ?

Berry-Sethi Approach: 1st step

empty$[r] = t$ if and only if $\epsilon \in [\![r]\!]$

... for example:



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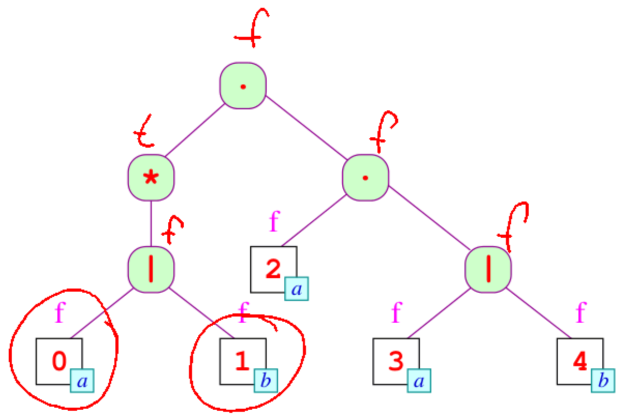

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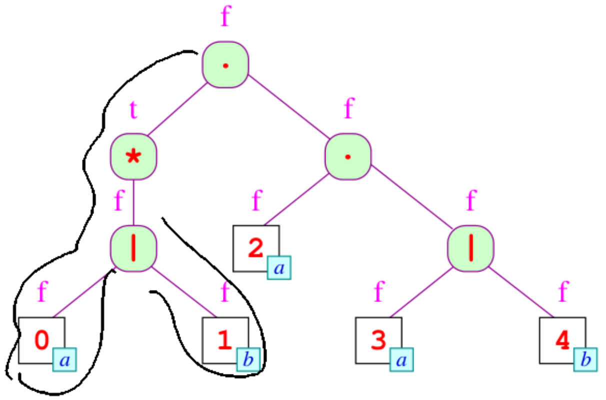

Berry-Sethi Approach: 2nd step

Implementation:
DFS post-order traversal

for leaves $r \equiv$ [i | x] we find empty$[r] = (x \equiv \epsilon)$.

Otherwise:

$$\begin{array}{lcl} \text{empty}[r_1 \mid r_2] & = & \text{empty}[r_1] \vee \text{empty}[r_2] \\ \text{empty}[r_1 \cdot r_2] & = & \text{empty}[r_1] \wedge \text{empty}[r_2] \\ \text{empty}[r_1^*] & = & t \\ \text{empty}[r_1?] & = & t \end{array}$$

Berry-Sethi Approach: 2nd step

The may-set of first reached read states: The set of read states, that may be reached from $\bullet r$ (i.e. while descending into r) via sequences of ϵ-transitions: first$[r] = \{i \text{ in } r \mid (\bullet r, \epsilon, \bullet \boxed{i \mid x}) \in \delta^*, x \neq \epsilon\}$

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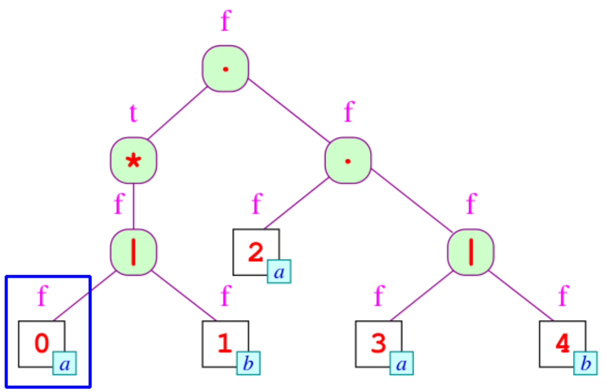

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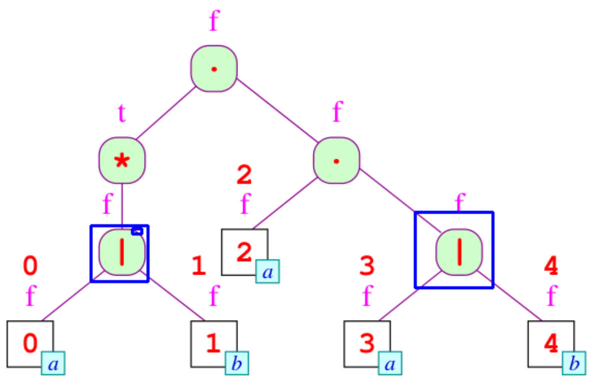

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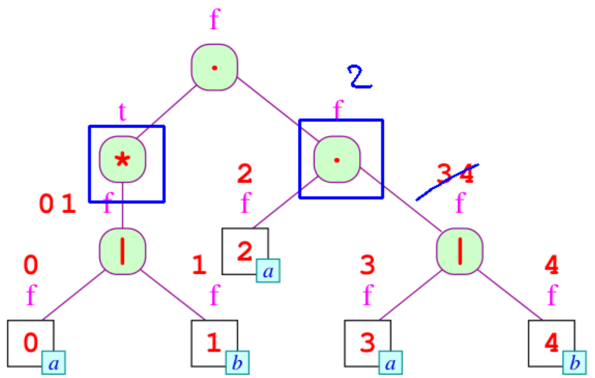

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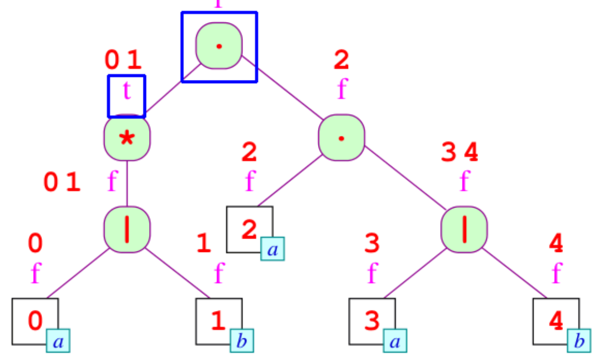

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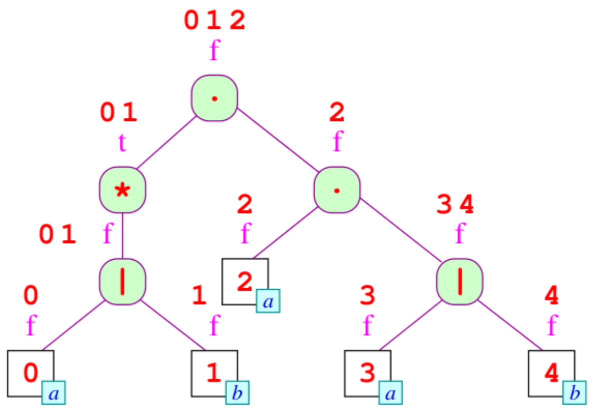

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Implementation:
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for leaves $r \equiv \boxed{i \mid x}$ we find first$[r] = \{i \mid x \neq \epsilon\}$.

Otherwise:

$$\text{first}[r_1 \mid r_2] = \text{first}[r_1] \cup \text{first}[r_2]$$

$$\text{first}[r_1 \cdot r_2] = \begin{cases} \text{first}[r_1] \cup \text{first}[r_2] & \text{if } \text{empty}[r_1] = t \\ \text{first}[r_1] & \text{if } \text{empty}[r_1] = f \end{cases}$$

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Berry-Sethi Approach: 3rd step

The may-set of next read states: The set of read states within the subtrees right of $r\bullet$, that may be reached next via sequences of ϵ-transitions. $\text{next}[r] = \{i \mid (r\bullet, \epsilon, \bullet\boxed{i \mid x}) \in \delta^*, x \neq \epsilon\}$

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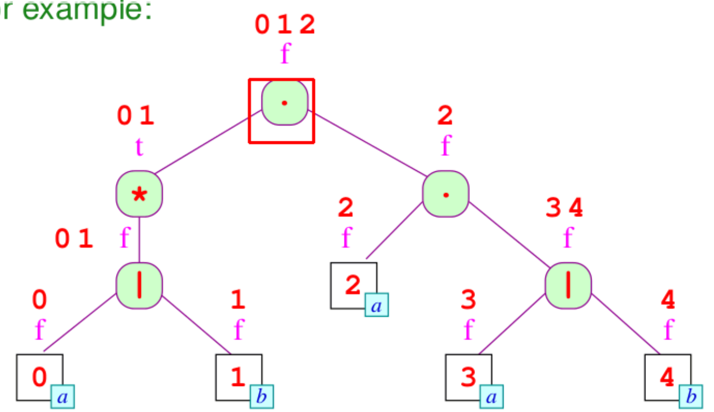

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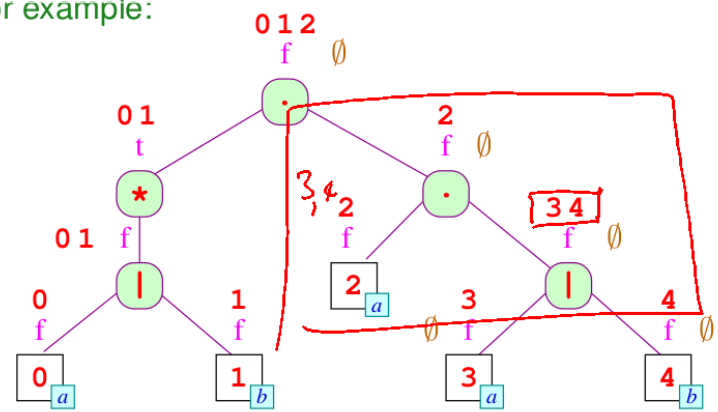

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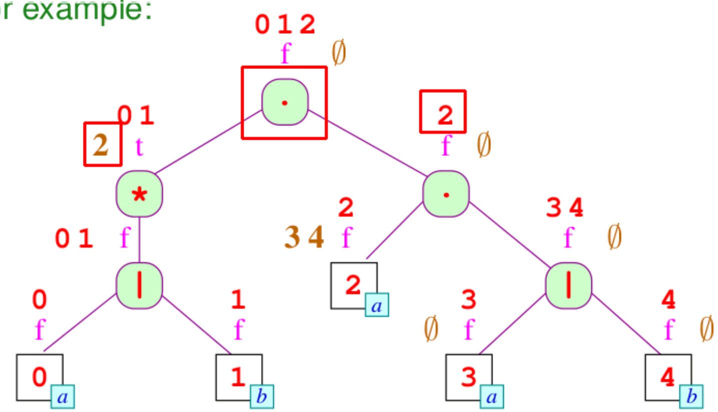

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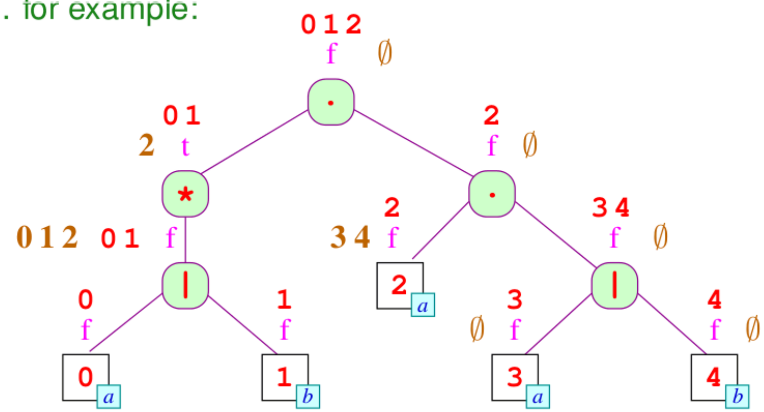

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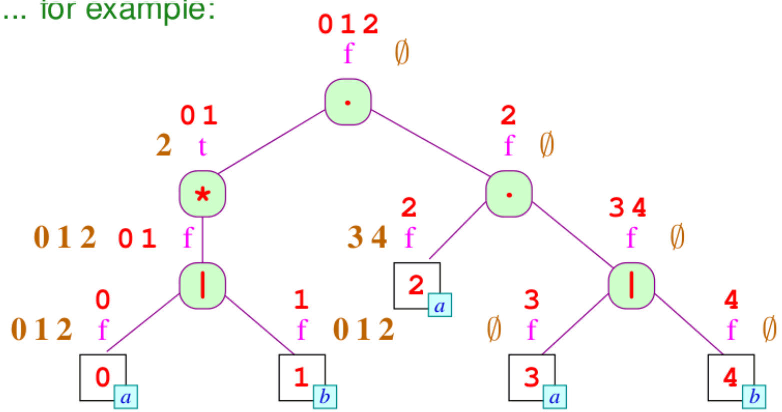

Berry-Sethi Approach: 3rd step

Implementation:
DFS pre-order traversal

For the root, we find: next$[e] = \emptyset$
Apart from that we distinguish, based on the context:

r	Equalities			
$r_1 \mid r_2$	next$[r_1]$	=	next$[r]$	
	next$[r_2]$	=	next$[r]$	
$r_1 \cdot r_2$	next$[r_1]$	=	first$[r_2] \cup$ next$[r]$	if empty$[r_2] = t$
			first$[r_2]$	if empty$[r_2] = f$
	next$[r_2]$	=	next$[r]$	
r_1^*	next$[r_1]$	=	first$[r_1] \cup$ next$[r]$	
$r_1?$	next$[r_1]$	=	next$[r]$	

Berry-Sethi Approach: 4th step

The may-set of last reached read states: The set of read states, which may be reached last during the traversal of r connected to the root via ϵ-transitions only: last$[r] = \{i \text{ in } r \mid (\boxed{i \mid x}\bullet, \epsilon, r\bullet) \in \delta^*, x \neq \epsilon\}$

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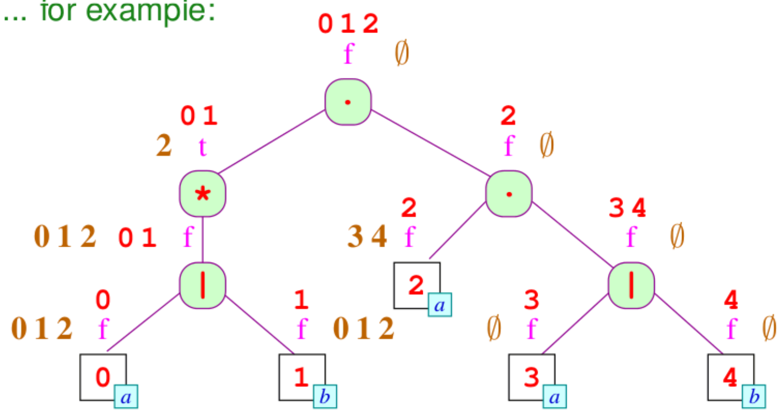

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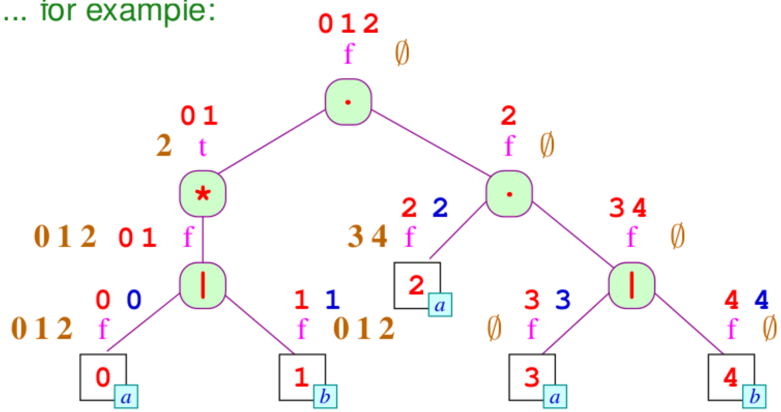

Berry-Sethi Approach: 4th step

Implementation:
DFS post-order traversal

for leaves $r \equiv$ [i | x] we find last$[r] = \{i \mid x \not\equiv \epsilon\}$.

Otherwise:

$$\begin{array}{lll} \text{last}[r_1 \mid r_2] & = & \text{last}[r_1] \cup \text{last}[r_2] \\ \text{last}[r_1 \cdot r_2] & = & \begin{cases} \text{last}[r_1] \cup \text{last}[r_2] & \text{if empty}[r_2] = t \\ \text{last}[r_2] & \text{if empty}[r_2] = f \end{cases} \\ \text{last}[r_1^*] & = & \text{last}[r_1] \\ \text{last}[r_1?] & = & \text{last}[r_1] \end{array}$$

Berry-Sethi Approach: (sophisticated version)

Construction (sophisticated version): Create an automanton based on the syntax tree's new attributes:

States: $\{\bullet e\} \cup \{i\bullet \mid i$ a leaf$\}$
Start state: $\bullet e$
Final states: last$[e]$ if empty$[e] = f$; $\{\bullet e\} \cup$ last$[e]$ otherwise
Transitions: $(\bullet e, a, i\bullet)$ if $i \in$ first$[e]$ and i labled with a.
$(i\bullet, a, i'\bullet)$ if $i' \in$ next$[i]$ and i' labled with a.

We call the resulting automaton A_e.

Berry-Sethi Approach

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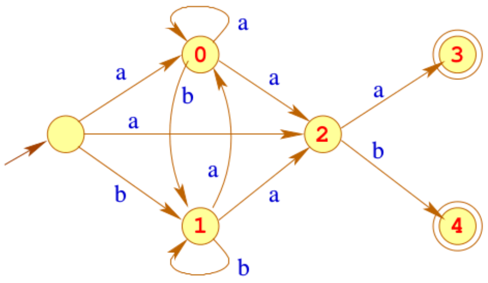
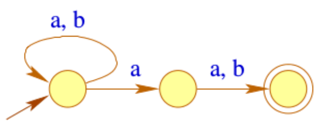


Remarks:

- This construction is known as Berry-Sethi- or Glushkov-construction.
- It is used for XML to define Content Models
- The result may not be, what we had in mind...

The expected outcome:



Remarks:

- ideal automaton would be even more compact
- but Berry-Sethi is rather directly constructed
- Anyway, we need a deterministic version

⇒ Powerset-Construction

Berry-Sethi Approach

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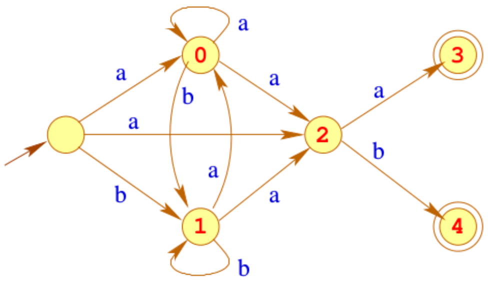

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Chapter 4:

Turning NFAs deterministic

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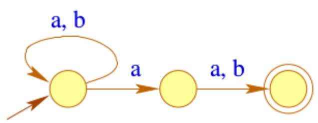

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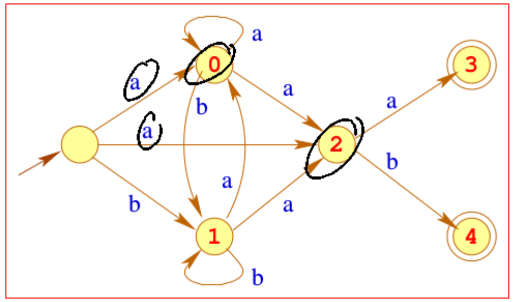

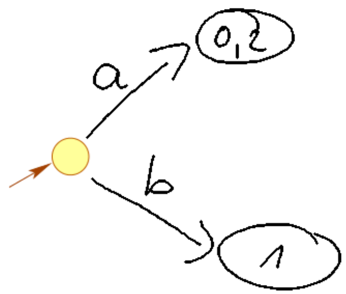

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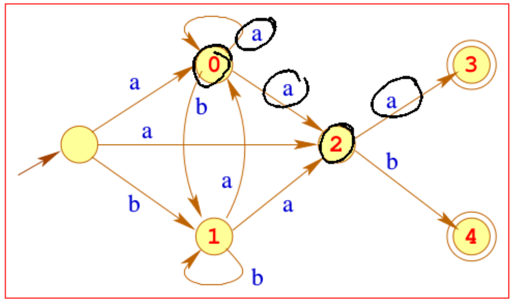

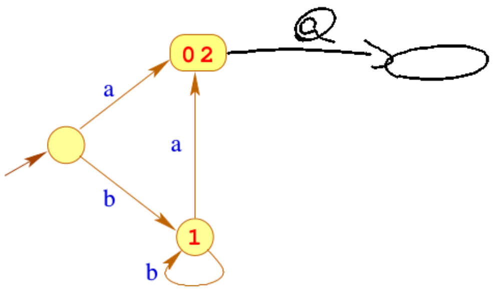

Powerset Construction

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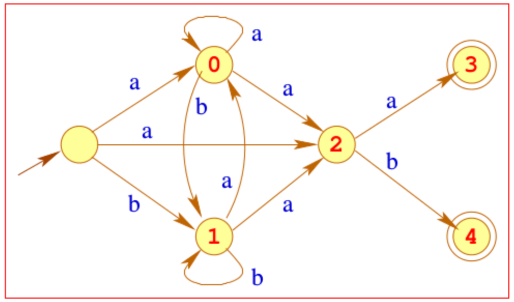

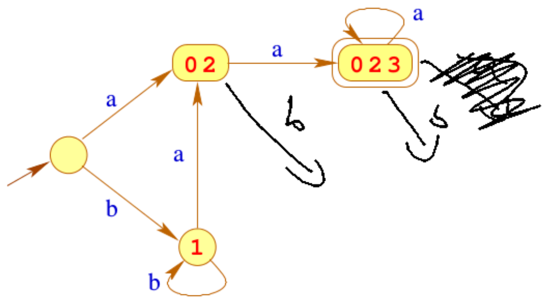

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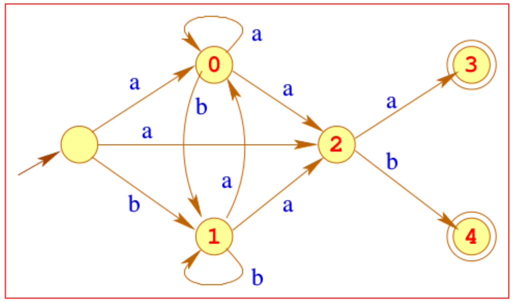

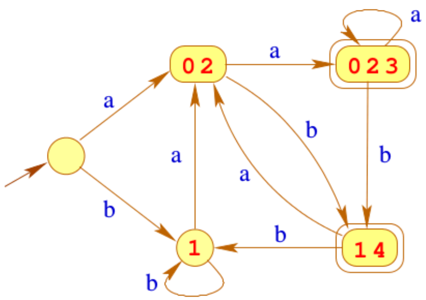

Powerset Construction

Theorem:

For every non-deterministic automaton $A = (Q, \Sigma, \delta, I, F)$ we can compute a deterministic automaton $\mathcal{P}(A)$ with

$$\mathcal{L}(A) = \mathcal{L}(\mathcal{P}(A))$$

Powerset Construction

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Construction:

States: Powersets of Q;

Start state: I;

Final states: $\{Q' \subseteq Q \mid Q' \cap F \neq \emptyset\}$;

Transitions: $\delta_{\mathcal{P}}(Q', a) = \{q \in Q \mid \exists p \in Q' : (p, a, q) \in \delta\}$.

Powerset Construction

Bummer!

There are exponentially many powersets of Q

- Idea: Consider only contributing powersets. Starting with the set $Q_{\mathcal{P}} = \{I\}$ we only add further states by need ...
- i.e., whenever we can reach them from a state in $Q_{\mathcal{P}}$
- Even though, the resulting automaton can become enormously huge
  ... which is (sort of) not happening in practice

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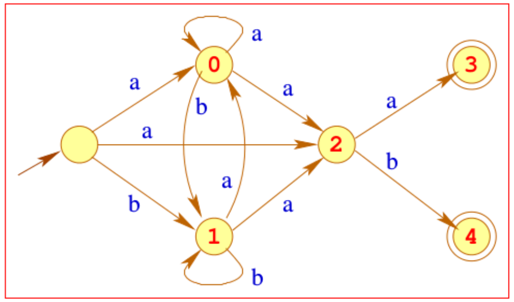
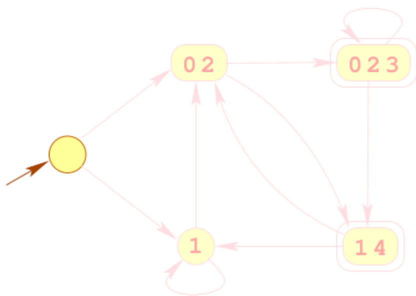
- Therefore, in tools like `grep` a regular expression's DFA is never created!
- Instead, only the sets, directly necessary for interpreting the input are generated while processing the input

Powerset Construction

... for example:

a	b	a	b





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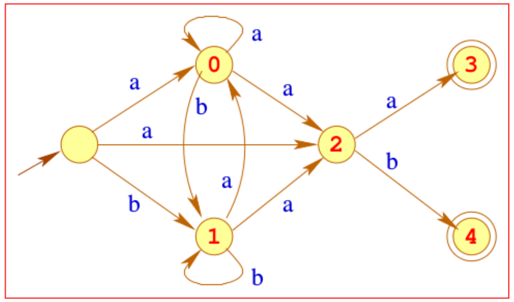

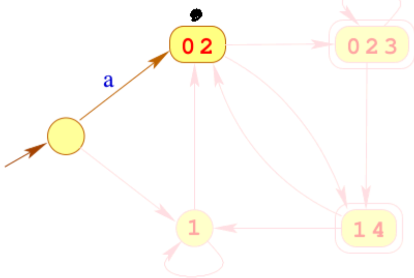

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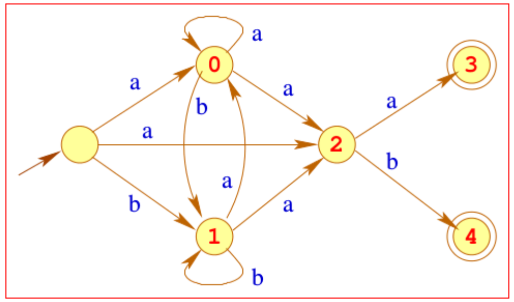

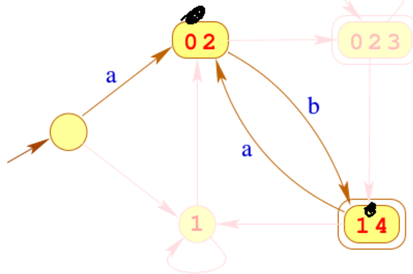

Powerset Construction

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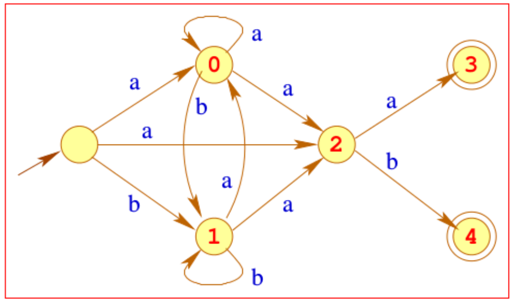

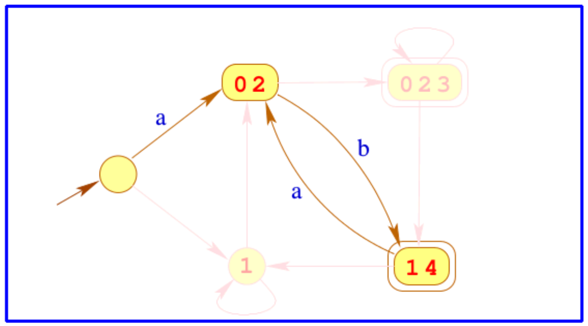

Remarks:

- For an input sequence of length n, maximally $\mathcal{O}(n)$ sets are generated
- Once a set/edge of the DFA is generated, they are stored within a hash-table.
- Before generating a new transition, we check this table for already existing edges with the desired label.

Powerset Construction

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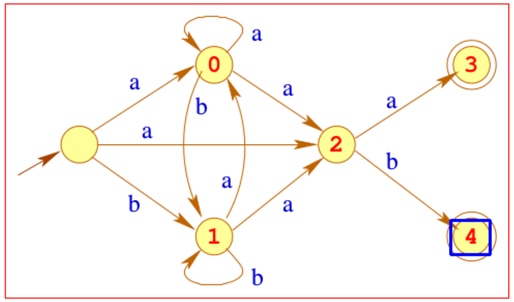

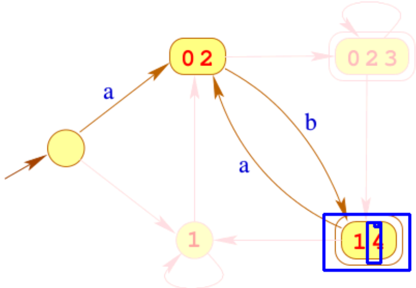

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Summary:

Theorem:

For each regular expression e we can compute a deterministic automaton $A = \mathcal{P}(A_e)$ with

$$\mathcal{L}(A) = [\![e]\!]$$

Chapter 5:
Scanner design