

## Script generated by TTT

Title: Petter: Compiler Construction (11.06.2020)  
- 28: LR2 to LR1

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Pages: 15

## What if precedences are not enough?

Example (very simplified lambda expressions):

$$\begin{aligned} E &\rightarrow (E)^0 | \text{ident}^1 | L^2 \\ L &\rightarrow \langle \text{args} \rangle \Rightarrow E^0 \\ \langle \text{args} \rangle &\rightarrow (\langle \text{idlist} \rangle)^0 | \text{ident}^1 \\ \langle \text{idlist} \rangle &\rightarrow \langle \text{idlist} \rangle \text{ident}^0 | \text{ident}^1 \end{aligned}$$

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$E$  rightmost-derives these forms among others:

$(\text{ident})$ ,  $(\text{ident}) \Rightarrow \text{ident}$ , ...  $\Rightarrow$  at least  $LR(2)$

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### Naive Idea:

poor man's  $LR(2)$  by combining the tokens  $)$  and  $\Rightarrow$  during lexical analysis into a single token  $)\Rightarrow$ .

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 \langle \text{idlist} \rangle &\rightarrow \langle \text{idlist} \rangle \text{ident}^0 | \text{ident}^1 \\
 E &\text{ rightmost-derives these forms among others:} \\
 (\underline{\text{ident}}), (\underline{\text{ident}}) \Rightarrow \text{ident}, \dots &\Rightarrow \text{at least } LR(2)
 \end{aligned}$$

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poor man's  $LR(2)$  by combining the tokens  $)$  and  $\Rightarrow$  during lexical analysis into a single token  $)\Rightarrow$ .

⚠ in this case obvious solution, but in general not so simple

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## What if precedences are not enough?

In practice,  $LR(k)$ -parser generators working with the lookahead sets of sizes larger than  $k = 1$  are not common, since computing lookahead sets with  $k > 1$  blows up exponentially. However,

- 1 there exist several practical  $LR(k)$  grammars of  $k > 1$ ,  
e.g. Java 1.6+ ( $LR(2)$ )
- 2 often, more lookahead is only exhausted locally
- 3 should we really give up, whenever we are confronted with a Shift-/Reduce-Conflict?

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### Theorem: $LR(k)$ -to- $LR(1)$

Any  $LR(k)$  grammar can be directly transformed into an equivalent  $LR(1)$  grammar.

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## LR(2) to LR(1)

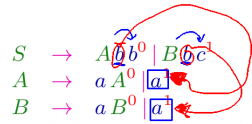
... Example:

$$\begin{aligned}
 S &\rightarrow Abb^0 | Bbc^1 \\
 A &\rightarrow aA^0 | a^1 \\
 B &\rightarrow aB^0 | a^1
 \end{aligned}$$

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# LR(2) to LR(1)

... Example:



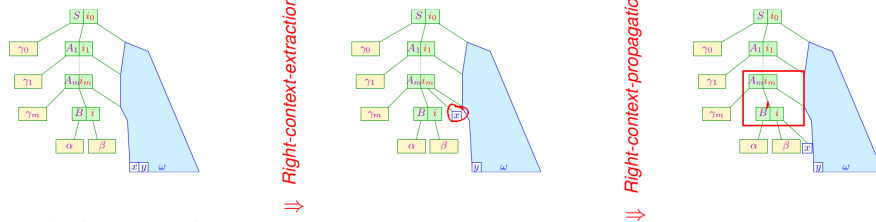
$S$  rightmost-derives one of these forms:

$$a^n abb, a^n abc, a^n a Abb, a^n a Bbc, Abb, Bbc \Rightarrow LR(2)$$

in  $LR(1)$ , you will have Reduce-/Reduce-Conflicts between the productions  $A, 1$  and  $B, 1$  under lookahead  $b$

# LR(2) to LR(1)

Basic Idea:



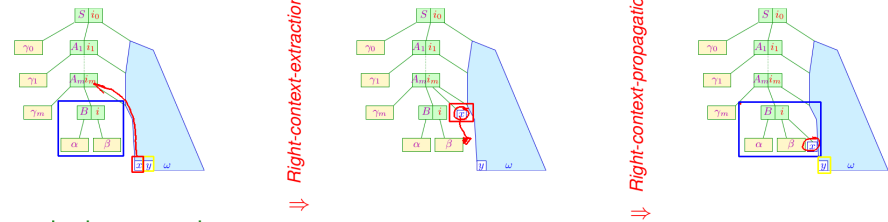
in the example:

Right-context is already extracted, so we only perform *Right-context-propagation*:

$$\begin{array}{l}
 S \rightarrow Abb^0 | Bbc^1 \\
 A \rightarrow aA^0 | a^1 \\
 B \rightarrow aB^0 | a^1
 \end{array}
 \Rightarrow
 \begin{array}{l}
 S \rightarrow \langle Ab \rangle b^0 | \langle Bb \rangle c^1 \\
 \langle A \rangle \rightarrow a \langle Ab \rangle | ab
 \end{array}$$

# LR(2) to LR(1)

Basic Idea:



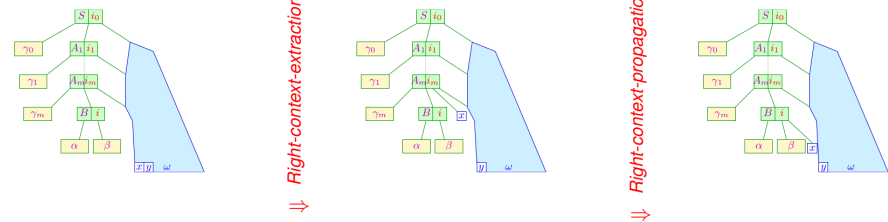
in the example:

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$$\begin{array}{l}
 S \rightarrow Abb^0 | Bbc^1 \\
 A \rightarrow aA^0 | a^1 \\
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 \end{array}
 \Rightarrow$$

# LR(2) to LR(1)

Basic Idea:



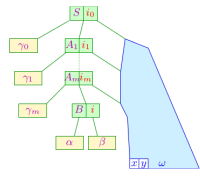
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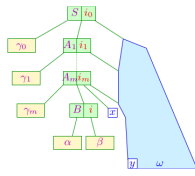
$$\begin{array}{l}
 S \rightarrow Abb^0 | Bbc^1 \\
 A \rightarrow aA^0 | a^1 \\
 B \rightarrow aB^0 | a^1
 \end{array}
 \Rightarrow
 \begin{array}{l}
 S \rightarrow \langle Ab \rangle b^0 | \langle Bb \rangle c^1 \\
 \langle Ab \rangle \rightarrow a \langle Ab \rangle | ab^1
 \end{array}$$

## LR(2) to LR(1)

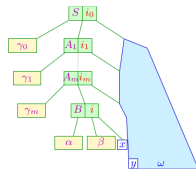
Basic Idea:



Right-context-extraction  $\Rightarrow$



Right-context-propagation  $\Rightarrow$



in the example:

Right-context is already extracted, so we only perform *Right-context-propagation*:

$$\begin{aligned} S &\rightarrow A b b^0 | B b c^1 \\ A &\rightarrow a A^0 | a^1 \\ B &\rightarrow a B^0 | a^1 \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} S &\rightarrow \langle A b \rangle b^0 | \langle B b \rangle c^1 \\ \langle A b \rangle &\rightarrow a \langle A b \rangle^0 | a b^1 \\ \langle B b \rangle &\rightarrow a \langle B b \rangle^0 | a b^1 \end{aligned}$$

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## LR(2) to LR(1)

Example cont'd:

$$\begin{aligned} S &\rightarrow \boxed{A'} b^0 | \boxed{B'} c^1 \\ A' &\rightarrow a A'^0 | a b^1 \\ B' &\rightarrow a B'^0 | a b^1 \end{aligned}$$

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## LR(2) to LR(1)

Example cont'd:

$$\begin{aligned} S &\rightarrow A' b^0 | B' c^1 \\ A' &\rightarrow a A'^0 | a b^1 \\ B' &\rightarrow a B'^0 | a b^1 \end{aligned}$$

$S$  rightmost-derives one of these forms:

$$a^n a b b, a^n a b c, a^n a A' b, a^n a B' c, A' b, B' c \Rightarrow LR(1)$$

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## LR(2) to LR(1)

Example 2:

$$\begin{aligned} S &\rightarrow b S S^0 \\ &| a^1 \\ &| a a c^2 \end{aligned}$$

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