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Title: Petter: Compiler Construction (04.06.2020) - LR(k) Grammars

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Pages: 20

LR(0)-Parser

Attention:
Unfortunately, the $LR(0)$-parser is in general non-deterministic.

We identify two reasons for a state $q \in Q$:

Reduce-Reduce-Conflict:

q: $A \to \gamma \bullet$, $A' \to \gamma' \bullet$ with $A \neq A' \vee \gamma \neq \gamma'$

Shift-Reduce-Conflict:

q: $A \to \gamma \bullet$, $A' \to \alpha \bullet a \beta$ with $a \in T$

Those states are called $LR(0)$-unsuited.

Revisiting the Conflicts of the LR(0)-Automaton

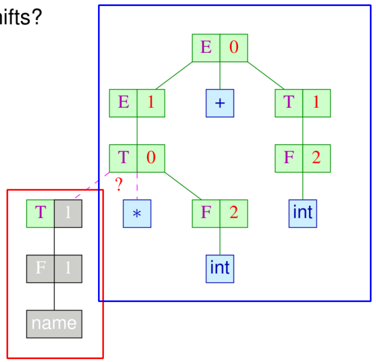
What differenciates the particular Reductions and Shifts?

Input:

$* \; 2 + 40$

Pushdown:

$(q_0 \; T)$



$$
\begin{array}{lcl}
E & \to & E + T \mid T \\
T & \to & T * F \mid F \\
F & \to & (\, E \,) \mid \text{int}
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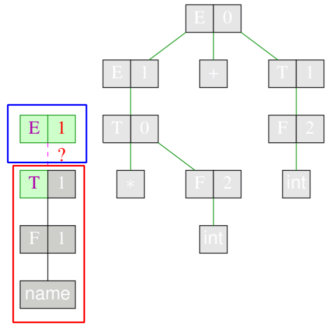

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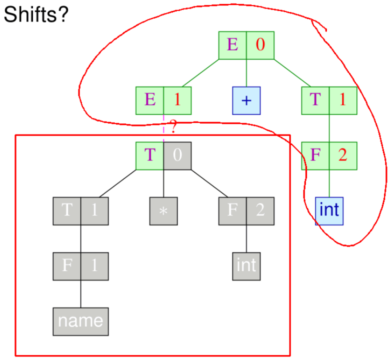

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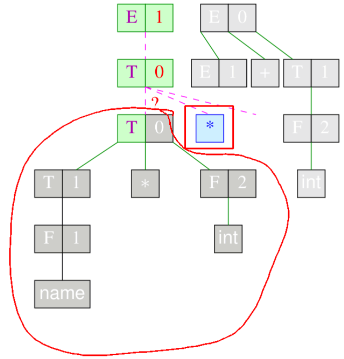

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Idea: In reverse rightmost derivations, *right context* determines derivations!

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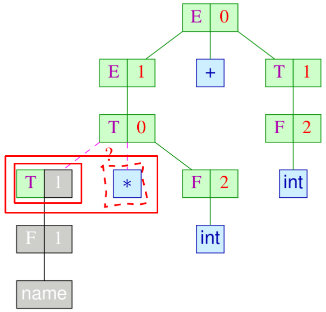

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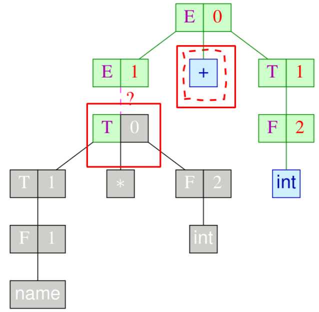

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LR(k)-Grammars

Idea: Consider k-lookahead in conflict situations.

Definition:
The reduced contextfree grammar G is called $LR(k)$-grammar, if
$\alpha\,\beta\,w|_{|\alpha\beta|+k} = \alpha'\,\beta'\,w'|_{|\alpha\beta|+k}$ with:

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Strategy for testing Grammars for $LR(k)$-property

1. Focus iteratively on all rightmost derivations $S \to_R^* \alpha\,X\,w \to \alpha\,\beta\,w$
2. Iterate over $k \geq 0$
   1. For each $\gamma = \alpha\,\beta w|_{|\alpha\beta|+k}$ (*handle with k-lookahead*) check if there exists a differently right-derivable $\alpha'\,\beta'\,w'$ for which $\gamma = \alpha'\,\beta'\,w'|_{|\alpha\beta|+k}$
   2. if there is none, we have found no objection against k being enough lookahead to disambiguate $\alpha\beta w$ from other rightmost derivations

LR(k)-Grammars

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Let $\;S \to_R^* \alpha\,X\,w \to \alpha\,\beta\,w\;$. Then $\;\alpha\,\underline{\beta}\;$ is of one of these forms:

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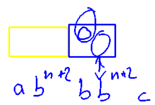

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(4) $S \to a\,A\,c \qquad A \to b\,A\,b \;|\; b$ ... is not $LR(k)$ for any $k \geq 0$:

Consider the rightmost derivations:

$$S \to_R^* a\,b^n\,A\,b^n\,c \to a\,b^n\,\underline{b}\,b^n\,c$$